INTERFERENCE ANALYSIS FOR WELLS PRODUCED AT CONSTANT PRESSURE

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INTERFERENCE ANALYSIS FOR WELLS PRODUCED AT CONSTANT PRESSURE

A

THESIS

Presented to the Faculty of the University of Alaska

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BY

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INTERFERENCE ANALYSIS FOR WELLS PRODUCED

AT CONSTANT PRESSURE

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ABSTRACT

Conventional interference tests are conducted with the active well producing at a constant rate. If the active well produces at a constant pressure, the flow rates are continuously changing. Hence, the pressure interference among constant pressure wells cannot be determined in the conventional way.

In this study, the rate interference among constant pressure production wells is modeled using the radial diffusivity equation. Pressure distributions in the reservoir for both constant pressure and constant rate couplets exhibit a similar zone of zero pressure gradient, which confirms the existence of an apparent no-flow boundary between any two constant pressure producers. Results show that a nearby fault boundary can be detected from flow rate versus time data.

The method given by Ramey[1] for evaluation of formation anisotropies in the constant rate environment does not fully hold for the constant pressure setting. Therefore, a new method for evaluating directional permeabilities was developed.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

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Jamal Akbar Ansari

INTRODUCTION

Conventional well test analysis methods have been developed for wells operated at a constant rate. Because of the production allowables, in some instances a well would be produced at an essentially constant rate throughout its life. However, as tighter reservoirs are being developed and as more wells are produced at capacity rates, reservoir fluids are discharged into a constant pressure separator or pipeline gathering system. In off-shore operations, constant pressure production is becoming more common than constant rate. In addition, geothermal steam wells are operated at constant pressure because the produced fluids are used to drive a back-pressured turbine. Finally, free flowing wells, including many water wells, operate at constant (atmospheric) pressure.

Ehlig-Economides[2,3,4] showed that rate transient information obtained from wells operated at constant pressure can be analyzed directly using methods analogous to conventional well tests. Ref. 2, 3 and 4 present a comprehensive description of the available methods for transient rate decline and pressure buildup analysis for wells produced at constant pressure. Constant pressure drawdown data is analyzed by constructing a semi-log graph of the reciprocal of the flow rate (1/q) versus the log of time. The reservoir permeability and the wellbore damage can be calculatd in a fashion similar to the constant rate case by using the slope of the semi-log straight line and the value of $(1/q)_{1\ hr}$. Analysis of pressure buildup data after constant pressure production is accomplished by following the conventional methods for pressure buildup analysis using the last measured flow rate in calculations requiring the flow rate.

This work is concerned with the interference effects among constant pressure wells. In a conventional interference test, a constant rate production in one well creates a pressure drawdown in an observation well that can be analyzed for reservoir properties. Multiple well testing has the inherent advantage of generally investigating more reservoir than a single well test.[6,7,8] Although it is a common belief that interference testing provides information about only the region between the wells, test results are actually influenced by a much larger region, as was shown by Vela and McKinley.[9]

Multiple well testing includes both drawdown interference and drawdown/buildup interference analysis. In the conventional drawdown interference well test, the flow rate at the active well is varied while bottom hole pressure response is measured at the observation wells. In the drawdown/buildup multiple well test, the bottom hole pressure response at the observation wells is measured both while the active well is flowing and after it is shut-in.

Type-curve matching is the standard method for analyzing both drawdown interference and drawdown/buildup interference data.

For the infinite acting radial flow geometry, the line source solution is used for analysis of drawdown interference. The drawdown/buildup interference data is analyzed using an extension of the line source solution type-curve which includes the dimensionless buildup for several given dimensionless flow times. Similar type-curves have been drawn for interference analysis in a predominantly linear flow geometry.[10]

To analyze an interference test by type-curve matching, the pressure data from the observation well is plotted on tracing paper as ΔP versus t on log-log coordinates using the same scale as that of the type-curve. The time and pressure matches are obtained by following the type-curve matching procedure given by Earlougher.[11]

A special form of multiple well testing is first described by Johnson, Greenkorn and Woods[12,13,14] as pulse testing. This technique uses a series of short rate pulses at the active well. Each pulse consists of a period of constant rate production or injection followed by a shut-in period. The same rate is maintained during each flowing period. The pressure response to the pulses is measured at the observation wells. The main advantage of the pulse test is its short duration. Lasting only a few hours, it causes less disruption to normal field operations than conventional interference tests which may require considerably more time.

A special application for interference testing invloves the determination of directional permeabilities. During the process of sedimentation, the grains forming the porous media tend to be oriented with their long axis parallel to the direction of the depositing current.[15] This mechanism may cause significant differences in the permeability of the porous media in different aerial directions. These directional permeability effects determine the optimum well configuration and spacing in a fully developed field.

Based on the work of Papadopulos[16], Ramey[1] presented a method for estimating directional permeabilities in an otherwise homogeneous reservoir from interference data.

Very little work has been done to devise methods for evaluating interference data when the active well(s) produces at a constant pressure instead of constant rate. Although Ehlig-Economides [2,3,4] provided tabular solutions for use in interference analysis, no practical method was presented. In this study, a new set of interference type-curves provide a simple method for analyzing the observed pressure at an observation well at an arbitrary distance from the constant pressure producer. The skin effect on the constant pressure producer is incorporated into the type-curves. In addition, a method for determining directional permeabilities from the pressure response in observation wells near a constant pressure producer is presented in the form of an example.

In the typical interference test, only one well is active. However, the interference effects among multiple producing wells provide information about the behavior of a well near a boundary or in a fully developed well pattern. For constant rate wells, the combined pressure behavior for several producers is determined using superposition in space of the pressure distribution for each individual well. To model interference among constant pressure wells, the continuously changing rates must be taken into account. Hence, superposition in both space and time must be applied.

In Ref. 4, Ehlig-Economides outlined a procedure for determining the theoretical rate response for interfering constant pressure producers but no results were provided. In this study, the procedure is used to determine the rate response and the pressure distribution for a pair of constant pressure producers. The results lead to a method for determining the location of a vertical fault near a constant pressure producer from the rate transients during production.

THEORY

The assumptions required to determine the transient rate response of a well produced at constant pressure are the same as the ones made to evaluate the transient pressure behavior of the wells operated at constant rate. The reservoir is assumed to be homogeneous and isothermal with constant thickness. If the formation is anisotropic, the permeability may vary in magnitude with direction but is constant in space and time. The reservoir fluid is only slightly compressible and has a constant viscosity. The flow in the reservoir is radial with negligible gravity effects and the pressure gradients are small everywhere in the reservoir. These assumptions establish the validity of the radial diffusivity equation:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\phi \mu c_t}{k}\frac{\partial p}{\partial t} \tag{1}$$

To model constant pressure production from a circular reservoir requires additional equations which represent the approximate initial and boundary conditions. For a reservoir initially at a constant pressure, p_i, the initial condition is given by:

$$p(r, o) = p_i \tag{2}$$

The inner boundary condition is specified as a constant flowing pressure which includes the skin effect:

$$p(r_w, t) = p_{wf} + s\left(r \frac{\partial p}{\partial r}\right)_{r = r_w} \tag{3}$$

The outer boundary condition for a well operating from the center of a circular reservoir of infinite extent is:

$$\lim_{r \to \infty} p(r, t) = p_i \tag{4}$$

In order to provide general solutions, dimensionless variables are defined as follows:

$$r_D = \frac{r}{r_w} \tag{5}$$

$$t_D = \frac{kt}{\phi \mu c_t r_w^2} \tag{6}$$

$$p_D(r_D, t_D) = \frac{p_i - p(r, t)}{p_i - p_{wf}} \tag{7}$$

$$q_D(t_D) = \frac{q(t)\mu}{2\pi kh(p_i - p_{wf})} \tag{8}$$

The resulting diffusivity equation in dimensionless variables is:

$$\frac{\partial^2 p_D}{\partial r_D^{\ 2}} + \frac{1}{r_D}\frac{\partial p_D}{\partial r_D} = \frac{\partial p_D}{\partial t_D} \tag{9}$$

The initial condition is:

$$p_D(r_D, 0) = 0 \tag{10}$$

The inner boundary condition is:

$$p_D(1, t_D) = 1 + s\left(\frac{\partial p_D}{dr_D}\right)_{r_D = 1} \tag{11}$$

The outer boundary condition is:

$$\lim_{r_D \to \infty} p_D(r_D, t_D) = 0 \tag{12}$$

Eqs. 10 to 3 completely describe the problem of a well producing at a constant wellbore pressure from the center of a circular reservoir under the assumptions listed above.

Interference Analysis

Tabular solutions for the pressure distribution around a constant pressure producer for several values of the wellbore skin factor were presented in Ref. 4. These were obtained by first solving, analytically, the diffusivity equation, with appropriate boundary and initial conditions in Laplace space. The results were then numerically inverted from Laplace space into real space using the Stehfest[18] algorithm.

The transient rate data, as has been mentioned by previous authors (Ref. 2), can be plotted versus dimensionless time to generate a type-curve for transient rate analysis.

To account for the skin effect, a dimensionless time, t_D', is defined in terms of the effective wellbore radius, r_w':

$$r_w' = r_w e^{-s} \tag{13}$$

and:

$$t_D' = \frac{kt}{\phi \mu c_t r_w^2 e^{-2s}} \tag{14}$$

When the dimensionless rate is graphed against t_D', a type curve results which includes the skin effect. For interference analysis, the tabulated solutions for the dimensionless pressure p_D, at a distance r_D from the producing well, are plotted versus t_D'/r_D^2 on log-log coordinates. The curves are indistinguishable for skin values ranging from -20 to +20. The solutions, however, are different for different values of the dimensionless radius, r_D, the distance between the operating well and the observation point. This difference in solutions, depending on the value of r_D, is also evident for constant rate wells when dimensionless pressure p_D is plotted versus t_D/r_D^2 on log-log coordinates, as was reported by Mueller and Witherspoon[19] and later on reproduced by Earlougher as Fig. C.1[11]. However, for the constant rate case, graphical solutions are indistinguishable for any value of t_D/r_D^2 greater than 20. This is so because the infinite solution applies for any value of r_D greater than 20 for constant rate operations. For constant pressure operations, the infinite solution does not hold for any value of r_D less than 8×10^4.

Based on the aforementioned solution, a set of type-curves of p_D versus t_D'/r_D^2 have been developed for several values of r_D. These type-curves are presented in Figs. 1 and 2 and the same information is presented in tabular form in Tables 1, 2 and 3 (Appendix - 2). These type-curves can be used to evaluate formation properties from an interference test. An example problem which illustrates the use of the interference type-curves for a constant pressure producer is

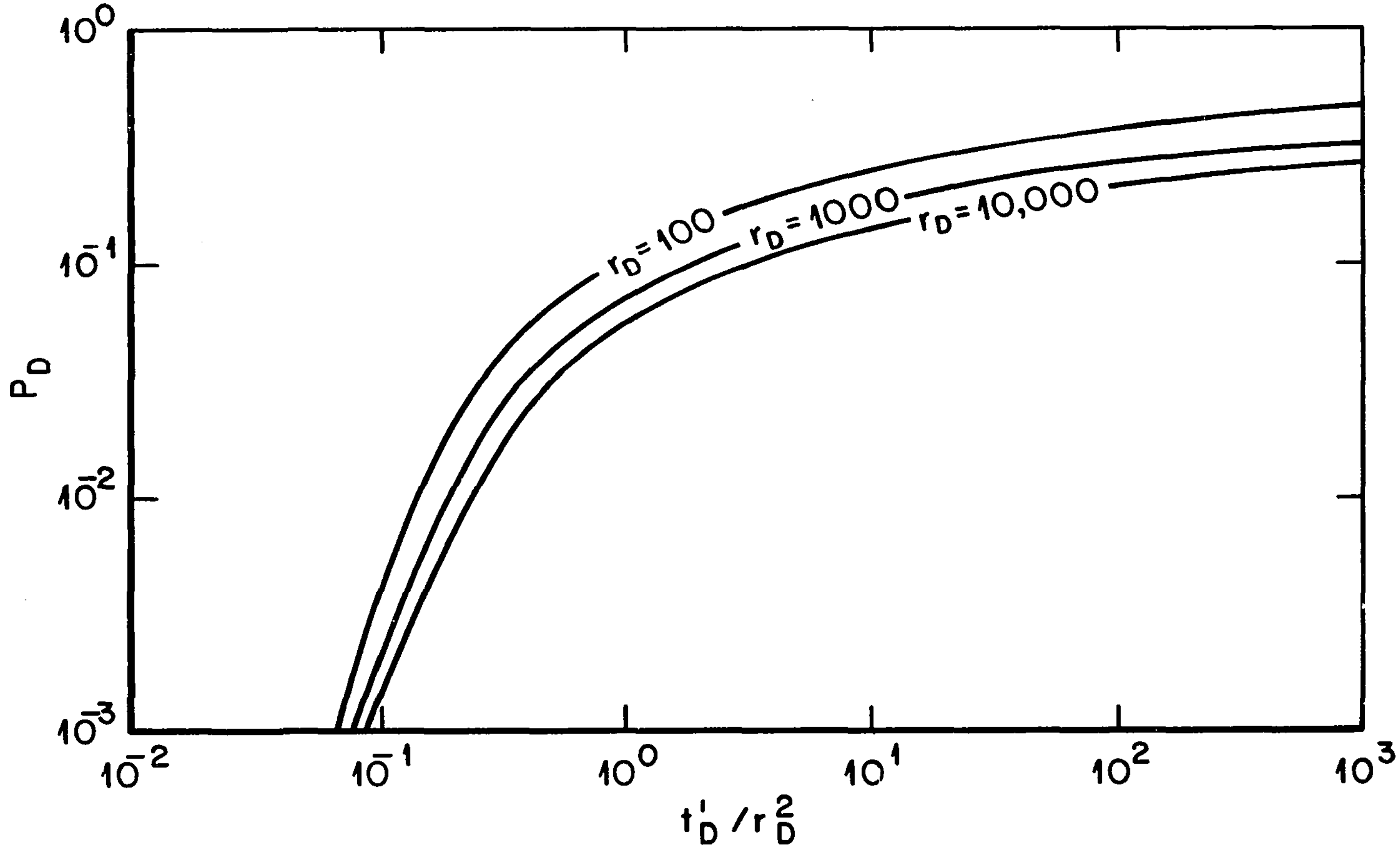


Figure 1. Dimensionless pressure for a single constant pressure producing well in an infinite system including the wellbore skin effect. (Early time behavior).

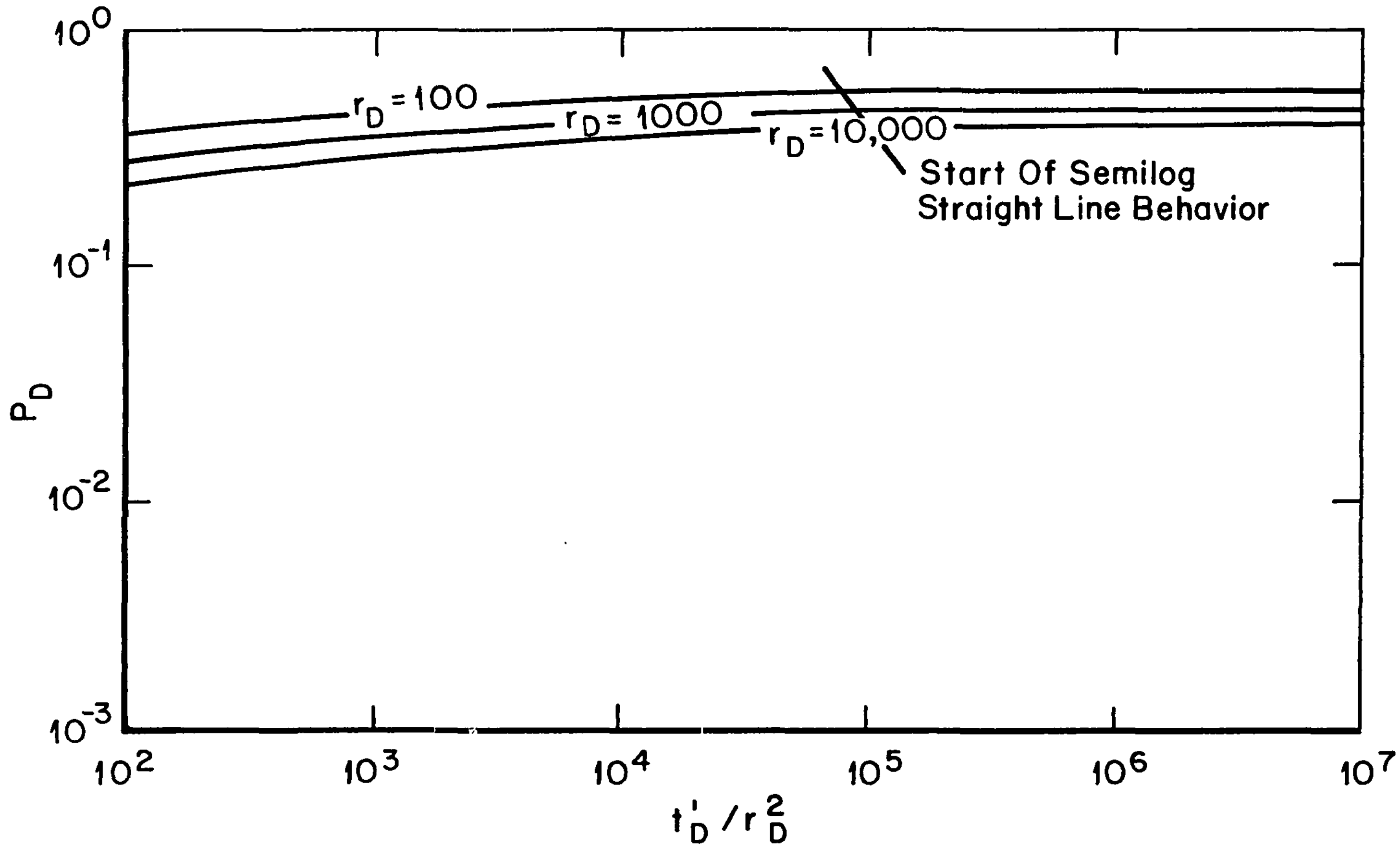


Figure 2: Dimensionless pressure for a single constant pressure producing well in an infinite system including the wellbore skin effect. (Late time response showing approach to the semi-log straight line behavior).

presented later in this study. A method for determining directional permeability effects is introduced in the same example.

Interference Effects Among Flowing Wells

The interference effects among wells operated at constant rate are evaluated in a straightforward manner using the principle of superposition in space. However, interference effects among wells operated at constant pressure cannot be determined in the conventional way because the rates are continuously changing. Hence, a different and independent method has been developed for the analysis of the interference effects among wells operated at constant pressure (Ref. 4). First, the continuously changing rates can be approximated in a stepwise manner analogous to the multiple rate well test analysis. Using superposition in time, the pressure change at observation Well A, due to constant pressure operation at Well B, a distance r from Well A, can be written as:

$$\Delta p_A(t) = -\frac{\mu}{2\pi kh}[q_o p_D\ (r,\ t) + (q_1 - q_o)\ p_D\ (r,\ t - t_1)$$

$$+ \cdots +$$

$$+ (q_n - q_{n-1})\ p_D\ (r,\ t - t_n)] \tag{15}$$

Now by rearranging eq. 15

$$\Delta p_A(t) = -\frac{\mu}{2\pi kh}\Bigg[q_o[p_D(r,\ t) - p_D(r,\ t - t_1)]$$

$$+ q_1\ [p_D(r,\ t - t_1) - p_D(r,\ t - t_2)]$$

$$+ \cdots +$$

$$+ q_n[p_D(r,\ t - t_{n-1}) - p_D(r,\ t - t_n)]\Bigg]$$

$$+ q_n p_n(t - t_n) \tag{16}$$

This rearrangement, which is similar to that used by van Everdingen and Hurst[20] for water influx modeling, isolates the flow rates for each time step. Next, the pressure difference in time, which multiplies each flow rate, is replaced by a partial derivative of pressure with respect to time, $(\frac{\partial p}{\partial t})\Delta t$.

The summation of stepwise rates in eq. 16 is replaced by an integral of continuously changing rates. The resulting expression is an exact solution for the pressure change at any point A due to constant pressure operation at Well B:

$$\Delta p_A(t) = \frac{-\mu}{2\pi kh} \int_0^t q_B(\tau) \frac{\partial p_D}{\partial \tau} (r, t - \tau) d\tau \qquad (17)$$

where:

τ = the variable of integration.

An expression similar to eq. 17, holds for effects from any number of constant pressure wells. Thus, a generalized expression can be written as:

$$p_i - p_A(x,y,t) = -\frac{\mu}{2\pi kh}\left[\int_o^t q_B(\tau)p_D'(r_B, t-\tau)d\tau\right.$$

$$+ \int_o^t q_C(\tau)p_D'(r_C, t-\tau)d\tau$$

$$+ \ldots +$$

$$\left. + \int_o^t q_Z(\tau)p_D'(r_Z, t-\tau)d\tau\right] \qquad (18)$$

where;

$$p_D' = \frac{\partial p_D}{\partial \tau} \qquad (19)$$

Eq. 18 is the superposition both in time and space of the pressure response from any number of wells for an observation well located at the point A and at time t.

Each integral in eq. 18 is in the form of the convolution integral. Therefore, making use of the convolution property, eq. 18 was simplified by taking the Laplace transform.

In order to determine the spacial and temporal pressure distributions from eq. 6, each rate function must be determined first. To do this, we must write equations for the pressure response at each constant pressure flowing well k:

$$(p_i - p_{wf})_k = -\frac{\mu}{2\pi kh}\left[\int_o^t q_k(\tau)p_{wD}'(t-\tau)d\tau\right.$$

$$+\sum_{i \neq k}\int_o^t q_i(\tau)p_D'(r_{ik}, t-\tau)d\tau$$

$$\left.+\sum_{j \neq k} q_j p_D(r_{jk}, t)\right] \qquad (20)$$

Taking the Laplace transform of eq. 20:

$$\frac{(p_i - p_{wf})_k}{\ell} = - \frac{\mu}{2\pi kh} \left[\bar{q}_k (\ell)\ \ell \bar{p}_{wD}(\ell) + \sum_{i \neq k} \bar{q}_i(\ell)\ \ell\ \bar{p}_D(r_{ik}, \ell) + \sum_{j \neq h} q_j \bar{p}_D(r_{jk}, \ell) \right] \quad (21)$$

The first term in eq. 21 represents the effects of the flowing well itself; the second term represents the effects of constant pressure wells which are operating in interference. The third term accounts for the constant rate wells which may be operating in interference with the rest. Also:

$$r_{ij} = [(x_i - x_j)^2 + (y_i - y_j)^2]^{0.5} \quad (22)$$

Eq. 21 can be written as:

$$[\Delta\bar{p}\ ]_n = [b]_{nxn} \times [\ \bar{q}\ ]_n \tag{23}$$

where:

$$\Delta\bar{p}_k = \frac{p_i - \bar{p}(x_k,\ y_k)}{\ell} - \frac{\mu}{2\pi kh} \sum_{i \neq k} \bar{p}_D(r_{ik},\ \ell) q_i \tag{24}$$

and:

$$b_{kj} = \begin{cases} -\frac{\mu}{2\pi kh}\,\bar{p}_D(r_{jk},\ \ell) \longrightarrow & k \neq j \\ \\ \ell\ \bar{p}_{wD}(\ \ell) \longrightarrow & k = j \end{cases} \tag{25}$$

The above represents a system of linear equations that can be solved analytically for the $\bar{q}_k$. Once the rate functions are known, the pressure distribution can be determined directly from eq. 18.

As an example, consider the case of two wells at a distance r_D from each other, both producing at constant pressure p_{w_f}. Then the pressure distribution can be computed from an expression similar to eq. 18:

$$p_i - p_{wf} = -\frac{\mu}{2\pi kh}\int_o^t q(\tau)p_{wD}'(t-\tau)d\tau$$

$$-\frac{\mu}{2\pi kh}\int_o^t q(\tau)p_D'(r_D, t-\tau)d\tau \qquad (26)$$

Rearranging, and using resulting dimensionless variables:

$$1 = - \int_{o}^{t} q_D(\tau) P_{wD}'(t_D - \tau) d\tau$$

$$- \int_{o}^{t} q_D(\tau) p_D'(r_D, t_D - \tau) d\tau \qquad (27)$$

because:

$$p_i - p_{wf} = \text{Constant}$$

and:

$$\frac{-\mu}{2\pi kh} = \text{Constant}$$

Applying Laplace transforms to eq. 27:

$$\frac{1}{\ell} = \ell \bar{q}_D(\ell) \bar{p}_{wD}(\ell) + \ell q_D(\ell) \bar{p}_D, (r_D, \ell) \qquad (28)$$

Eq. 28 is solved for $\bar{q}_D(\ell)$, to obtain the rate function:

$$\bar{q}_D(\ell) = \frac{1}{\ell^2 [\bar{p}_{wD}(\ell) + \bar{p}_D(r_D, \ell)]} \tag{29}$$

Substituting known Laplace space solutions for p_{wD} and p_D:

$$\bar{q}_D(\ell) = \frac{K_1(\sqrt{\ell})}{\sqrt{\ell}[K_o(\sqrt{\ell}) + s\sqrt{\ell}K_1(\sqrt{\ell})_1 + K_o(r_D\sqrt{\ell})]} \tag{30}$$

Using the Stehfest[18] algorithm, a real space solution for $q_D(t_D)$ can be obtained by numerically inverting the Laplace space equivalent obtained from eq. 30.

Once the rate functions are known, the pressure distribution can be determined in terms of the dimensionless pressure, p_D:

$$\bar{p}_D = \bar{q}_D(\ell)[p_D(r_1, \ell) + p_D(r_2, \ell)]\ell \tag{31}$$

where the dimensionless pressure is defined as in eq. 7. In the next section, the implications of the above formulation will be clarified.

PRACTICAL APPLICATIONS

Determination of No-Flow/Fault Boundary

To compare the pressure distribution between two constant pressure flowing wells to the pressure distribution between two constant rate producers, the pressure profiles for both constant pressure and constant rate couplets were calculated. The information thus obtained, is presented in graphical form in Figs. 3 and 4.

The pressure contour maps (Figs. 3 & 4) for both constant rate and constant pressure production are qualitatively similar. The zone of zero pressure gradient bisecting the two wells confirms the existence of an apparent no-flow boundary for constant pressure operations analogous to that present for the constant rate couplet.

To examine the rate transients resultant from the presence of a nearby constant pressure producer, the rates were calculated for the set of constant pressure Wells A and B operating at a specified distance from each other (Fig. 5). The rate transient data thus obtained for Well A, were plotted as reciprocal of rate (1/q) versus log of time (log t). When plotted this way, the data lie on a semi-log straight line, which doubles in slope when the effect of

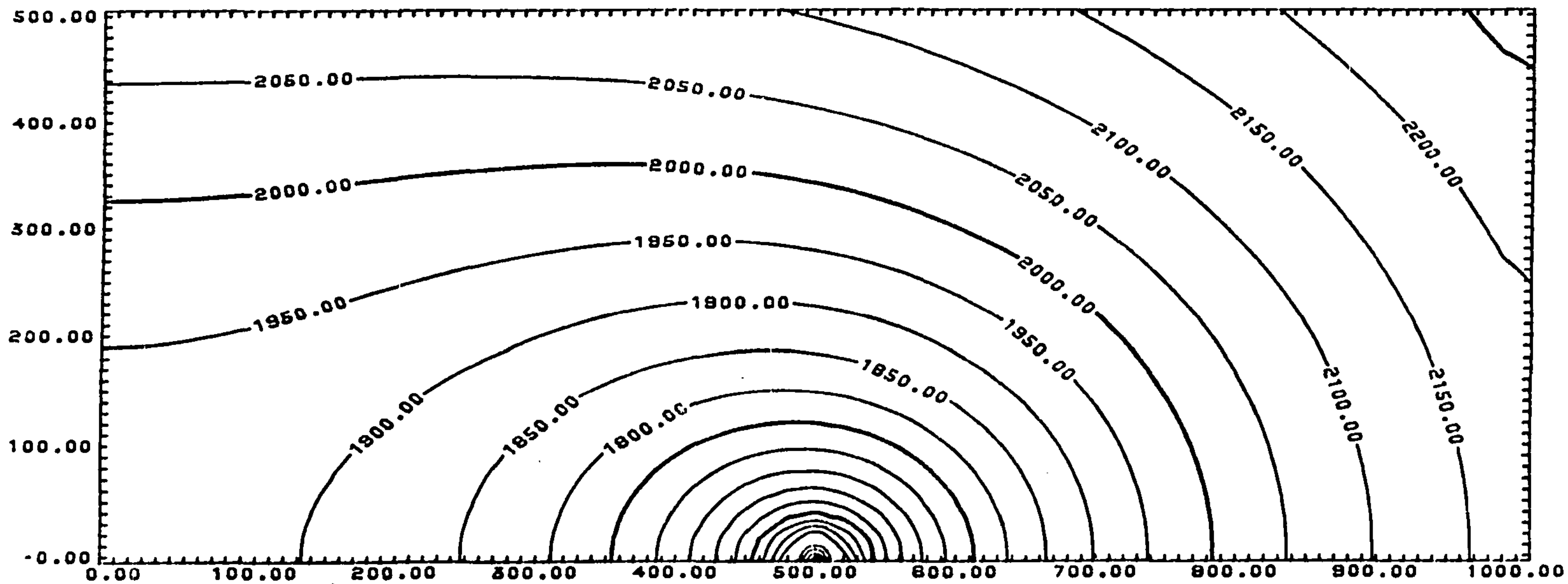


Figure 3: Pressure profile in the reservoir around a constant pressure producer flowing in interference with another constant pressure producer.

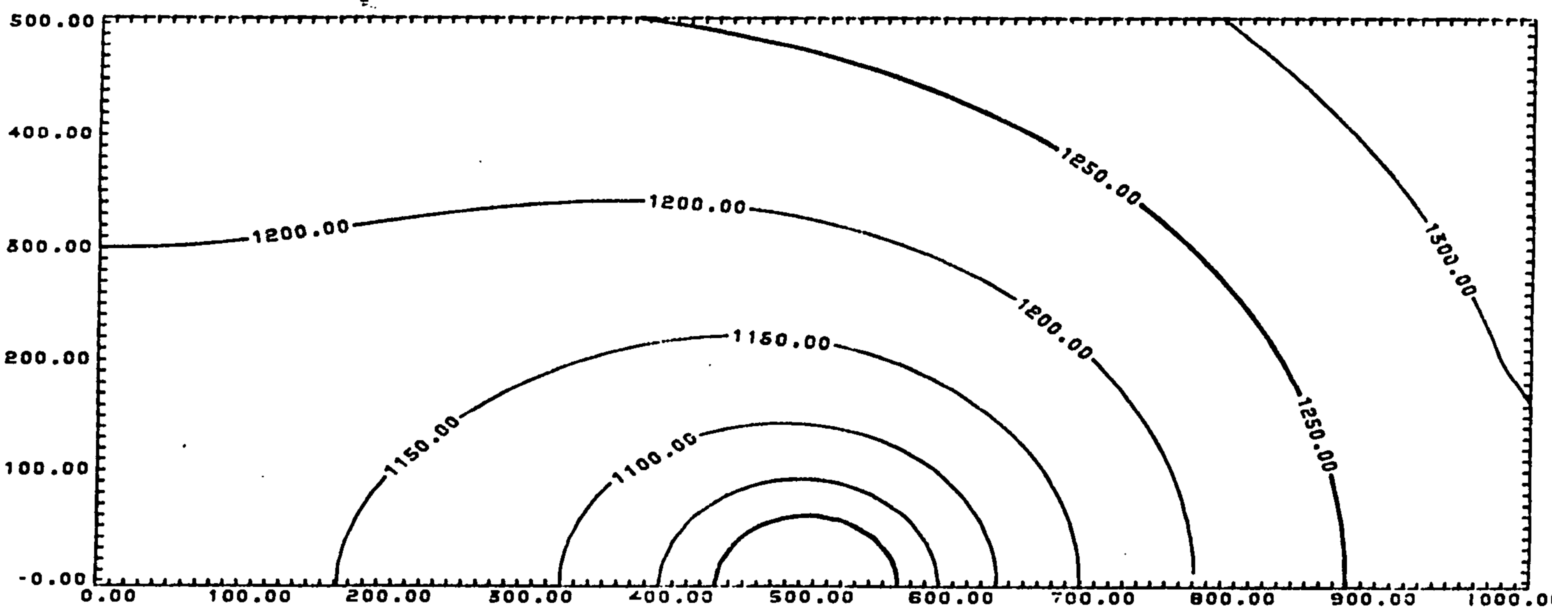


Figure 4: Pressure profile in the reservoir around a constant rate producer flowing in interference with another constant rate producer.

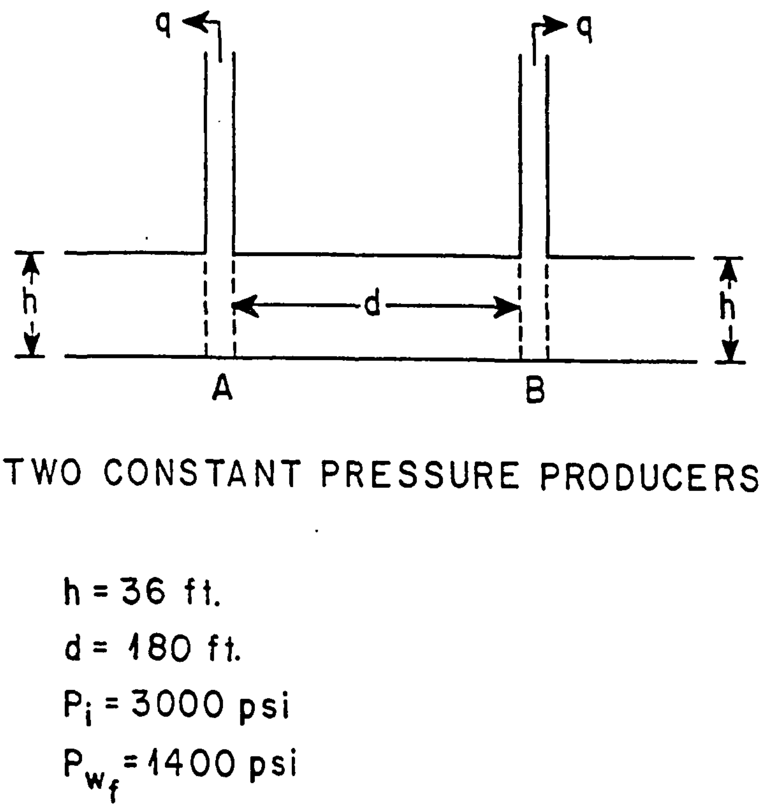


Figure 5: Two constant pressure producers operating at a distance, d, from each other.

production at Well B is sensed at Well A (Fig. 6). The time t_x at which the slope doubles, depends on the spacing between the two wells:

$$t_x = \frac{1688.\phi\mu c_t d^2}{k} \quad (32)$$

Fig. 7 shows the increase in the time at which the slope doubles as the spacing between the wells flowing in interference increases.

The doubling of the slope of the semi-log straight line may be due either to the production at a neighboring well or to an actual no-flow boundary caused by a nearby fault. If the doubling of the slope is due to a fault boundary, then the distance 'L' from the producing well to the fault is given by a rearrangement of eq. 32:

$$L = 0.01217\left(\frac{kt_x}{\phi\mu c_t}\right)^{0.5} \quad (33)$$

Eq. 33 is identical to the relationship given by Earlougher[11], for constant rate wells. To illustrate the interpretation, the following example is presented.

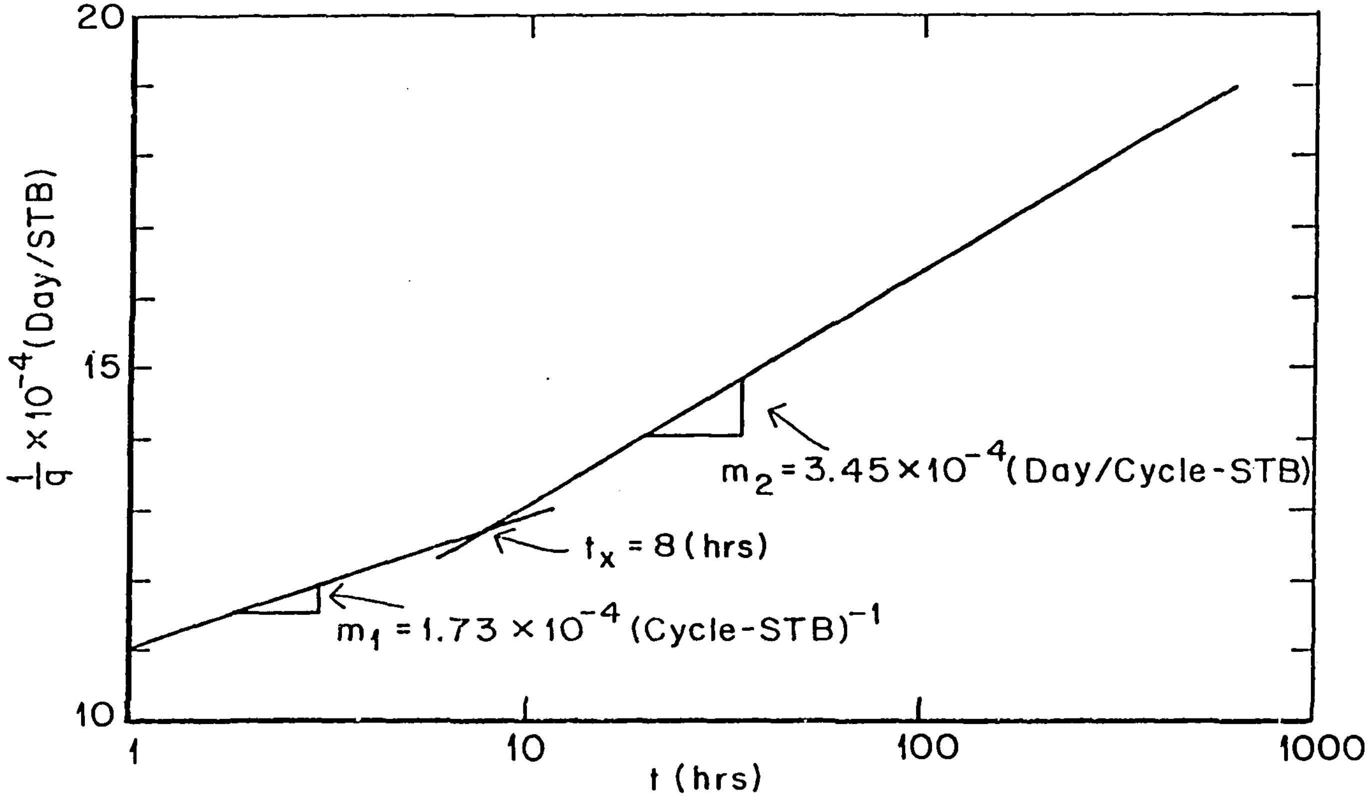


Figure 6: Semi-log graph of the rate transients for a constant pressure producer showing the slope doubling effect of the interference from a nearby constant pressure producer. (μ = 3.0 cp, c_t = 10 x 10^{-6}, r_w = 0.5 ft., ϕ = 0.25, $p_i - p_{wf}$ = 1600 psi)

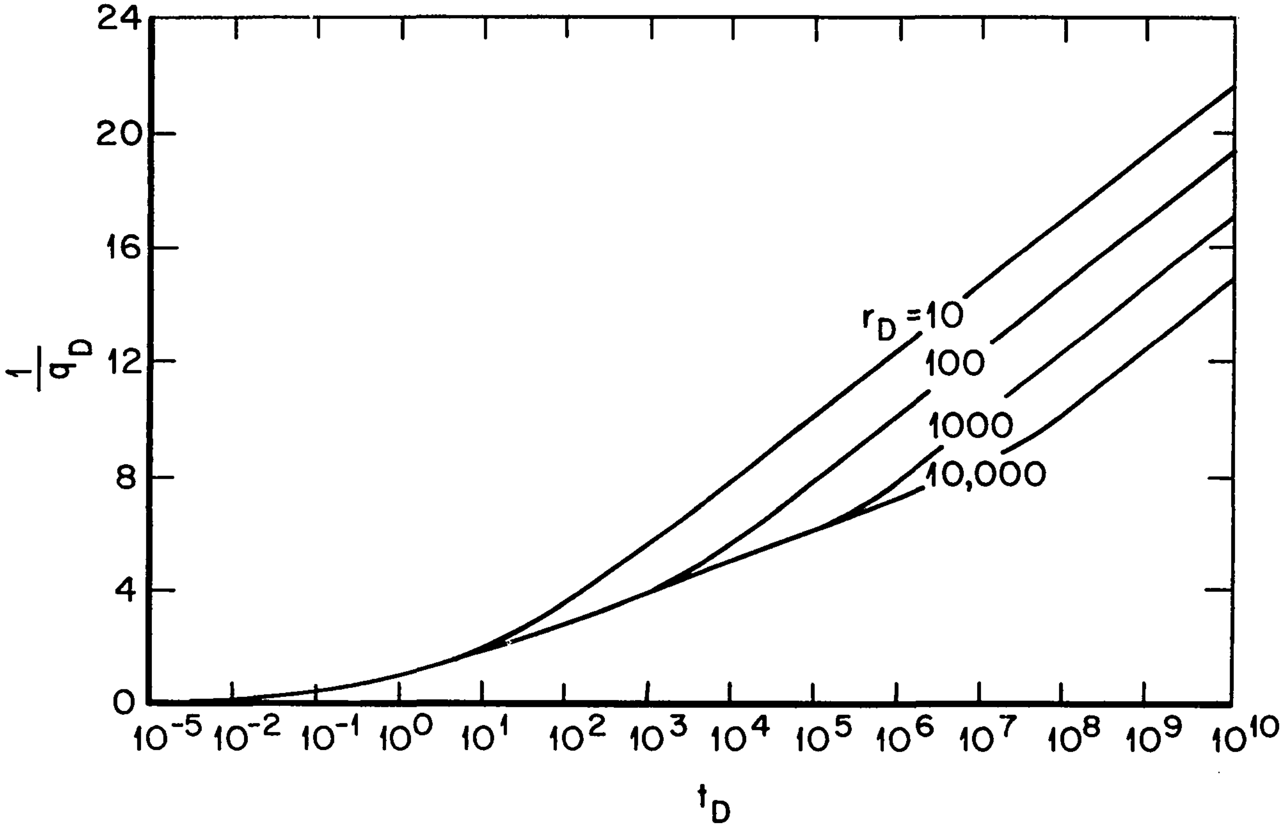


Figure 7: Semi-log graph of the rate transients for one of two interfering constant pressure producers showing the time at which the slope doubles as a function of the distance between the two flowing wells.

Example 1: Detection of a nearby fault from transient rate data.

The rate transient data for Well C, along with known reservoir rock and fluid parameters are given in Table 4.

As indicated by Earlougher[11], the formation permeability (k) is evaluated using the slope of the semi-log straight line before the interference effects are apparent:

$$k = \frac{162.6 B\mu}{mh(p_i - p_{wf})} \tag{34}$$

$$m = 1.65 \times 10^{-4} \text{ - from Fig. 8}$$

$$k = \frac{(162.6 \times 1.315 \times 3.1)}{1.65 \times 10^{-4} \times 36 \times (3000-1400)}$$

$$\underline{k = 70 \text{ md}}$$

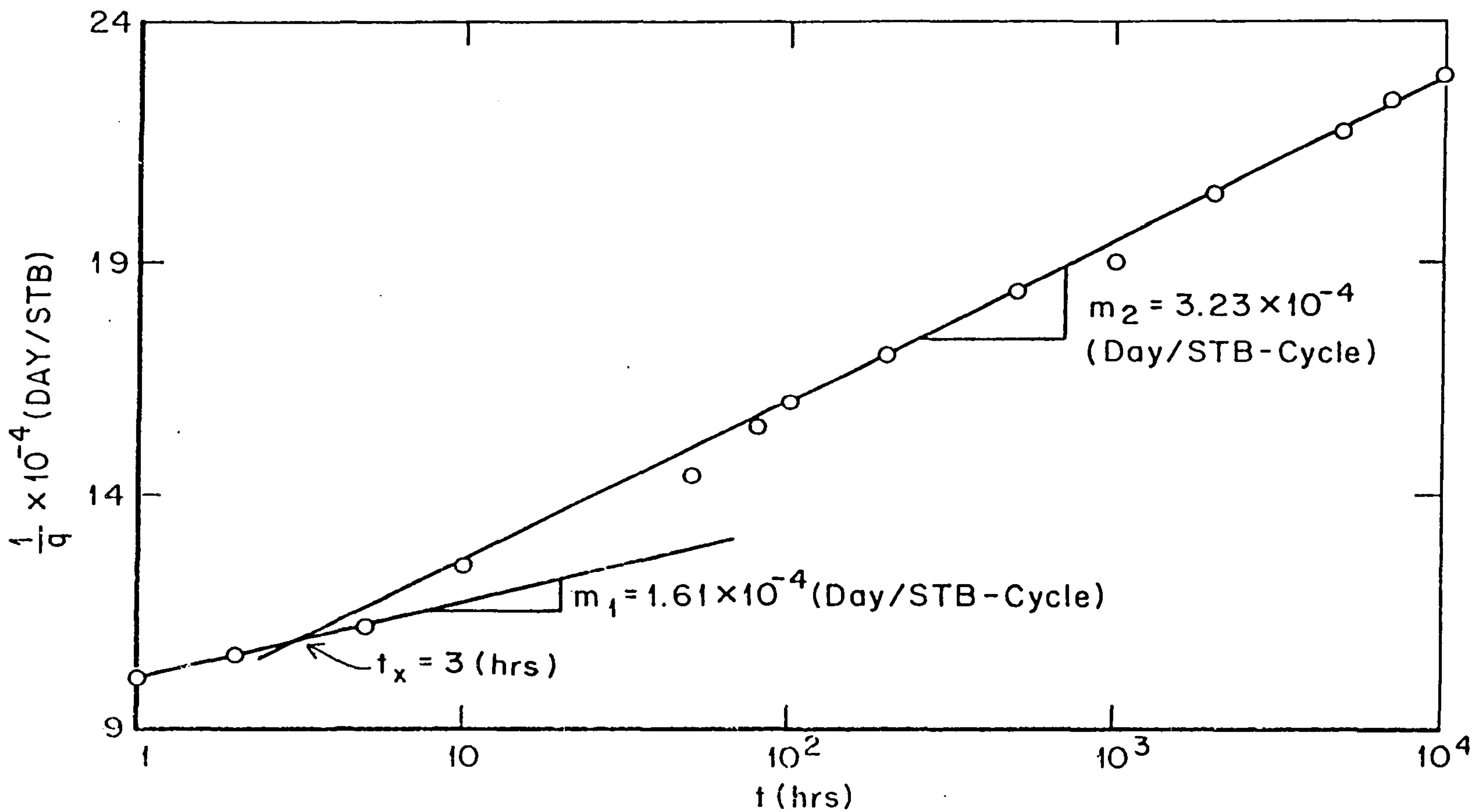


Figure 8: Semi-log plot of the rate transient data, for Example 1, with the doubling of the slope confirmed the presence of a nearby fault boundary.

The wellbore skin factor is determined from the value of $(\frac{1}{q})_{1\ hr}$ obtained from Fig. 8:

$$s = 1.1513\ \left[\frac{(\frac{1}{q})_{1\ hr}}{m} - \log\left(\frac{k}{\phi\mu c_t r_w^2}\right) + 3.2275\right] \tag{35}$$

$$(\frac{1}{q})_{1\ hr} = 10.0 \times 10^{-4} \quad \text{- from Fig. 8}$$

$$s = 1.1513\ \left[\ \frac{10.0 \times 10^{-4}}{1.61 \times 10^{-4}}\right.$$

$$\left. - \log\left(\frac{70}{0.2 \times 3.1 \times 10 \times 10^{-6} \times (0.5)^2}\right) + 3.2275\right]$$

$\underline{s = +2.2}$

The slope of the semi-log straight line doubles after 3 hours (Fig. 8), which indicates the presence of a nearby fault boundary. The distance at which this fault occurs from Well C, can be calculated using eq. 33:

$$L = 0.4295\left(\frac{70 \times 3.}{0.2 \times 3.1 \times 10 \times 10^{-6}}\right)^{0.5}$$

$L = \underline{71.\ ft}$

Determination of Directional Permeabilities

Evaluation of formation anisotropy, using the pressure transient data from one constant rate flowing well and two observation wells given by Ramey[1], can be qualitatively adapted to the constant pressure environment. The differences between the two arise because of the definition of the dimensionless variables p_D and t_D':

The dimensionless variables for constant pressure operations as presented by Earlougher[11] and used by Ramey[1] are:

$$p_D(r_D,\ t_D) = \frac{kh}{141.2qB\mu}\ [p_i - p(r,\ t)] \tag{36}$$

and;

$$t_D = \frac{0.000264kt}{\phi\mu c_t r_w^2} \tag{37}$$

The dimensionless variables for a constant pressure environment as developed in this study are:

$$p_D(r_D,\ t_D) = \frac{p_i - p(r,\ t)}{p_i - p_{wf}} \tag{7}$$

and;

$$t_D' = \frac{0.000264kt}{\phi\mu c_t r_w^2 e^{-2s}} \tag{14}$$

The dimensionless pressure p_D for constant pressure flowing wells is a pressure ratio and does not include reservoir rock and fluid properties. The reason for this is the continuously changing rates under constant wellbore pressure instead of the continuously changing pressure that results from constant rate operations. Due to this difference, the pressure match obtained from type-curve matching of constant pressure interference data does not provide a means to calculate reservoir permeability k.

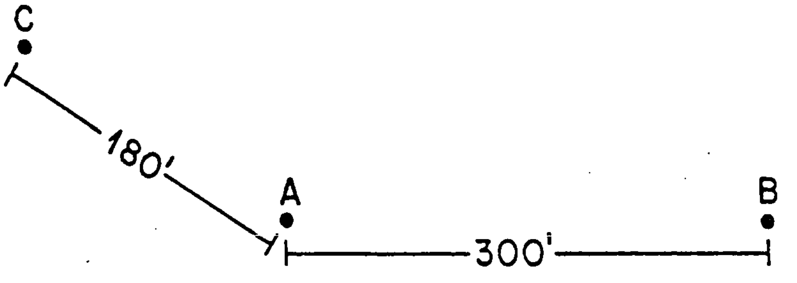
The difference in the definition of the dimensionless time t_D' for constant pressure setting is that it includes the well bore skin effect, s, at the active well, where as the dimensionless time t_D for constant rate flowing wells does not. The wellbore skin effect must be included because the response time at the observation well is dependent on the skin effect at the constant pressure producer. In constant rate wells, the effect of skin on the response at an observation well is negligible. Since the pressure match cannot be used to determine the permeability, the time match is used. As a result, the analysis cannot be done without knowledge of the porosity and total compressibility of the formation. The value of the wellbore skin factor is also required.

In a constant pressure interference well test designed to evaluate reservoir anisotropies, the rate transient data at the active well is also analyzed. This compensates for the lost information due to the different definitions of the dimensionless pressure p_D and obtains the well bore skin effect which is required to calculate permeability from the time match.

Example 2: Determination of directional permeabilities.

Computer simulated rate and pressure data were generated for a constant pressure producer Well A and two observation Wells B and C. The location of the three wells is shown in Fig. 9 and the formation and fluid properties in Tables 5, 6 & 7 (Appendix - 3).

As a first step, the rate transient data for Well A was plotted as 1/q versus log of time (Fig. 10), which resulted in a semi-log straight line. Using eq. 33, the slope (m) of the semi-log straight line gives the geometric mean permeability ($\bar{k}$) at the producer:



Well#	Status	Location	
		X	Y
A	Producer *	0	0
B	Observation	300.0	0.0
C	Observation	-125.0	125.0

*Constant Pressure

Figure 9: Well location and status for the interference well test in Example 2.

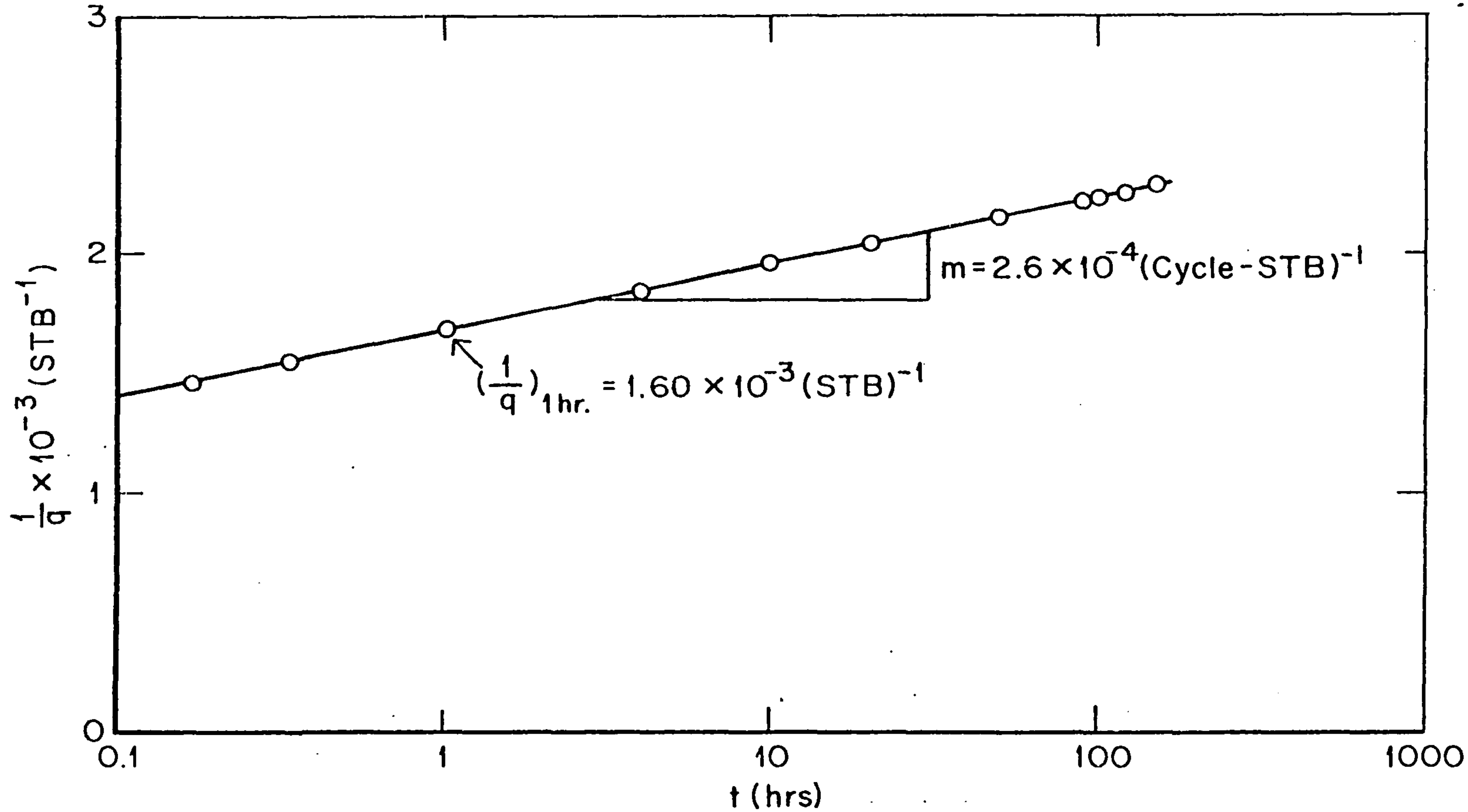


Figure 10: Semi-log plot of the rate transients for the producing well in Example 2.

$$\bar{k} = \frac{(162.6 \times 1.315 \times 3.1)}{(2.6 \times 10^{-4} \times 60.4 \times 2000)}$$

$$\underline{\bar{k} = 20.4 \text{ md}}$$

Once the slope (m) and permeability ($\bar{k}$) are known, the value of $(\frac{1}{q})_{1\ hr.}$ is obtained from the plot of 1/q versus log of time (Fig. 10) and the value of skin is obtained using eq. 35:

$$s = 1.1513 \left[\frac{1.60 \times 10^{-3}}{2.6 \times 10^{-4}} \right.$$

$$\left. - \log \left(\frac{20.4}{0.25 \times 3 \times 13 \times 10^{-6} \times (.6)^2} \right) + 3.2275 \right]$$

$$\underline{s = +3.05}$$

Next, the pressure transient data from Wells B and C were graphed as Δp versus time (t) on tracing paper (Figs. 11 and 12 respectively), using the log-log scale that matches the scale on the type-curve (Fig. 1). A time and pressure match was obtained for both observation wells.

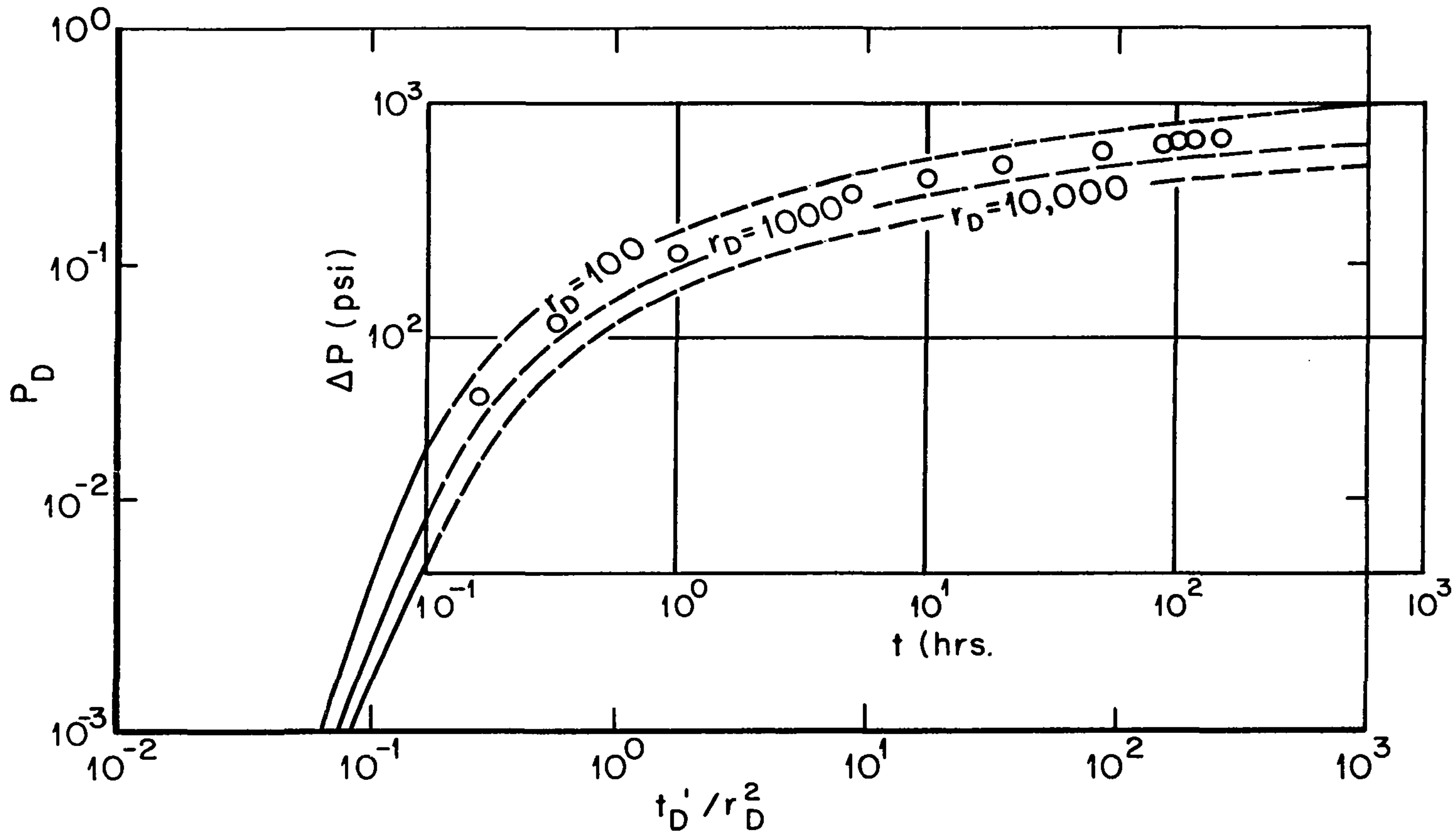


Figure 11: Type-curve matching for the pressure versus time data observed at Well B.

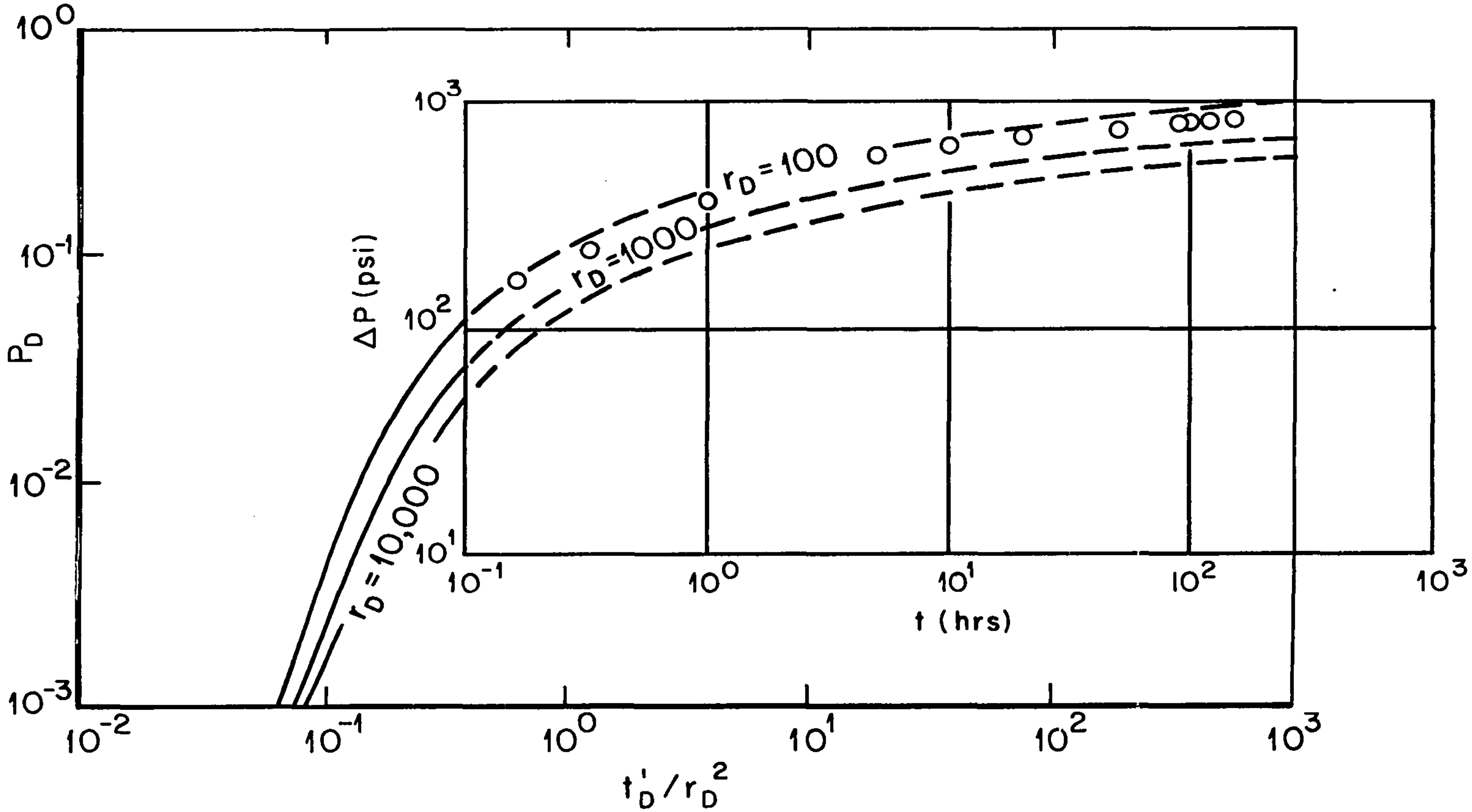


Figure 12: Type-curve matching for the pressure versus time data observed at Well C.

The time match for Well B was:

$$\frac{t_D'}{r_D^2} = 17 \quad @ \quad t = 10 \text{ hrs.}$$

and for Well C:

$$\frac{t_D'}{r_D^2} = 38 \quad @ \quad t = 10 \text{ hrs.}$$

These values, along with the values of other constants, were then substituted into eq. 38:

$$\frac{t_D'}{r_D^2} = \frac{0.0002637t}{\phi\mu c_t e^{-2s}} \quad \frac{k_{xx}k_{yy} - k_{xy}^2}{k_{xx}Y^2 + k_{yy}X^2 - 2k_{xy}XY} \tag{38}$$

From eq. 38, eq. 39 and eq. 40 were obtained for Well B and Well C respectively:

$$k_{yy} = -\frac{k_{xy}^{\ 2}}{(13.18 - k_{xx})} \tag{39}$$

and

$$k_{xx} = \frac{-11.44\ k_{xy} - k_{xy}^{\ 2} - 5.72k_{yy}}{5.72 - k_{yy}} \tag{40}$$

The third equation comes from the permeability calculated from the rate data at the producing well.

By definition:

$$\bar{k} = (k_{xx}k_{yy} - k_{xy}2)^{0.5} \tag{41}$$

where $\bar{k}$ is the geometric mean permeability which we have already calculated from the rate transient analysis of Well A. Thus, solving for k_{xy}, the third equation is:

$$k_{xy} = (k_{xx}k_{yy} - 416.2)^{0.5} \tag{42}$$

Now there are the three equations required to solve for the three unknowns k_{xx}, k_{yy} and k_{xy}. The three equations we have are non-linear. Therefore, a small computer program based on the Newton Raphson iteration for nonlinear equations was developed; (Appendix 4) and the values of the three components of the permeability tensors were calculated:

$$k_{xx} = 19.8 \text{ md.}$$

$$k_{yy} = 30.0 \text{ md.}$$

$$k_{xy} = 13.3 \text{ md.}$$

Once the values for all three permeability components have been determined, the maximum principal permeability k_{XX} and the minimum principal permeability k_{YY} can be calculated using eq. 43 and 44, as were given by Ramey[1] for constant rate operations.

$$k_{XX} = \frac{1}{2} \ (k_{xx} + k_{yy}) + [(k_{xx} - k_{yy})^2 + 4k_{xy}{}^2]^{0.5} \tag{43}$$

$$k_{XX} = \underline{39.14 \text{ md.}}$$

$$k_{YY} = \frac{1}{2} \ (k_{xx} + k_{yy}) - [(k_{xx} - k_{yy})^2 + k_{xy}{}^2]^{0.5} \tag{44}$$

$$k_{YY} = \underline{10.71 \text{ md.}}$$

Finally, the angle between the maximum principal permeability axis 'X' and our orientation axis 'x' can be calculated using the following equation given, by Ramey[1].

$$\theta = \arctan \left(\frac{k_{XX} - k_{xx}}{k_{xy}}\right) \tag{45}$$

$$\underline{\theta = 55.45^{\circ}}$$

CONCLUSIONS

The interference effects among wells operated at constant pressure can be determined using superposition both in time and space. The solutions provided in this study confirm the existence of a no-flow boundary among wells operated at constant pressure similar to the ones that are known to exist among wells operated at constant rate. The presence of a no-flow or a nearby fault boundary can be determined in a simple constant pressure drawdown well test, analogous to the constant rate case, as doubling of the slope of the semi-log straight line, when inverse of rate (1/q) is graphed versus log of time (log t). The distance to the nearby fault or a no-flow boundary can be calculated using eq. 21, in a manner similar to the constant rate operations.

The reservoir anisotropies in a constant pressure environment can be evaluated using interference type-curves in a fashion somewhat similar to conventional methods for determining directional permeabilities in a constant rate setting.

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NOMENCLATURE

A = area, L^2

c_t = total compressibility, Lt^2/m

d = distance between flowing wells, L

D = wellbore diameter, L

g_c = units conversion factor

h = reservoir thickness, L

I_0, I_1 = Modified Bessel functions

k = reservoir absolute permeability, L^2

$\bar{k}$ = reservoir geometric mean permeability, L^2

k_{XX} = maximum principal permeability, L^2

k_{YY} = minimum principal permeability, L^2

k_{xx}; k_{yy}; k_{xy} = components of permeability tensors, L^2

K_0, K_1 = Modified Bessel functions

L = distance to the fault, L

m = slope of $1/q$ vs. log t graph for a constant-pressure test, t/L^3

p = pressure, m/Lt^2

p^* = extrapolated pressure on Horner buildup graph, m/Lt^2

p_D = dimensionless pressure ratio, $\dfrac{p_i - p_{r,t}}{p_i - p_{wf}}$

p_{wD} = dimensionless wellbore pressure, $2\pi kh(p_i - p_{wf})/q\mu$

p_i = initial reservoir pressure, m/Lt^2

p_{wf} = flowing bottom-hole pressure, m/Lt^2

p_{ws} = bottom-hole pressure after shut-in, m/Lt^2

q = production rate, L^3/t

q_D = dimensionless production rate, $\frac{q\mu}{2\pi kh(p_i - p_{wf})}$

$(1/q)_{1\ hr}$ = ordinate value at 1 hour on straight-line graph of $1/_q$ vs. log t, t/L^3

r_D = dimensionless radius, r/r_w

r_e = reservoir radius, L

r_{eD} = dimensionless reservoir radius, r_e/r_w

r_w = wellbore radius, L

r_w' = effective wellbore radius, $r_w e^{-s}$, L

ℓ = Laplace space variable

s = skin factor

t = time

t_D = dimensionless time in terms of effective well base radius, $r_w' = \frac{kt}{\phi\mu c_t r_w^2 e^{-2s}}$

t_p = production time, t

t_x = time at which the slope of the semi-log straight line doubles, t

x,y = coordinates in well system, L

X = maximum principal permeability axis, L

Y = minimum principal permeability axis, oriented at 90^o to X axis, L

ϕ = porosity

θ = angle between x and X axis, positive in counterclockwise direction from x-axis

μ = fluid viscosity, m/Lt

$\bar{\rho}$ = average wellbore fluid density, M/L^3

τ = variable of integration

APPENDIX - 1

UNITS CONVERSIONS

Variable	Darcy, SI Metric Units	Oilfield Units
t_D	$\frac{kt}{\phi\mu c_t r_w^2}$	$\frac{.000264\ kt}{\phi\mu c_t r_w^2}$
q_D	$\frac{qB\mu}{2\pi kh(p_i-p_{wf})}$	$\frac{141.2\ qB\mu}{kh(p_i-p_{wf})}$
m	$\frac{.1832\ qB\mu}{kh}$	$\frac{162.2\ qB\mu}{kh}$
c	atm^{-1}, Pa^{-1}	psi^{-1}
h	cm,m	ft
k	darcy, m^2	md
p	atm, Pa	psi
q	cm^3/sec, m^3/sec	barrels/day
r	cm, m	ft
t	sec, sec	hr
μ	cp, Pa-sec	cp

APPENDIX - 2

TABULATED SOLUTIONS

TABLE 1

Tabulated solutions for p_D versus t_D'/r_D^2 for a single constant pressure well in an infinite system including wellbore skin effect, from -20 to +20

$r_D = 100$

0.10000E-01	0.95641E-06
0.20000E-01	0.30173E-05
0.30000E-01	0.61292E-05
0.40000E-01	0.20426E-04
0.50000E-01	0.17954E-03
0.60000E-01	0.55793E-03
0.70000E-01	0.11785E-02
0.80000E-01	0.20321E-02
0.90000E-01	0.30937E-02
0.10000E 00	0.43333E-02
0.20000E 00	0.21286E-01
0.30000E 00	0.38924E-01
0.40000E 00	0.54422E-01
0.50000E 00	0.67761E-01
0.60000E 00	0.79307E-01
0.70000E 00	0.89422E-01
0.80000E 00	0.98366E-01
0.90000E 00	0.10635E 00
0.10000E 01	0.11356E 00
0.20000E 01	0.16123E 00
0.30000E 01	0.18841E 00
0.40000E 01	0.20710E 00
0.50000E 01	0.22119E 00
0.60000E 01	0.23244E 00
0.70000E 01	0.24176E 00
0.80000E 01	0.24968E 00
0.90000E 01	0.25655E 00
0.10000E 02	0.26261E 00
0.20000E 02	0.30041E 00
0.30000E 02	0.32092E 00
0.40000E 02	0.33480E 00
0.50000E 02	0.34519E 00
0.60000E 02	0.35346E 00
0.70000E 02	0.36029E 00
0.80000E 02	0.36610E 00
0.90000E 02	0.37114E 00
0.10000E 03	0.37558E 00
0.20000E 03	0.40333E 00
0.30000E 03	0.41847E 00
0.40000E 03	0.42876E 00
0.50000E 03	0.43650E 00
0.60000E 03	0.44267E 00
0.70000E 03	0.44778E 00
0.80000E 03	0.45213E 00
0.90000E 03	0.45592E 00

0.10000E 04	0.45926E 00
0.20000E 04	0.48027E 00
0.30000E 04	0.49183E 00
0.40000E 04	0.49972E 00
0.50000E 04	0.50568E 00
0.60000E 04	0.51044E 00
0.70000E 04	0.51440E 00
0.80000E 04	0.51778E 00
0.90000E 04	0.52072E 00
0.10000E 05	0.52332E 00
0.20000E 05	0.53975E 00
0.30000E 05	0.54884E 00
0.40000E 05	0.55509E 00
0.50000E 05	0.55981E 00
0.60000E 05	0.56360E 00
0.70000E 05	0.56675E 00
0.80000E 05	0.56944E 00
0.90000E 05	0.57179E 00
0.10000E 06	0.57387E 00
0.20000E 06	0.58706E 00
0.30000E 06	0.59440E 00
0.40000E 06	0.59946E 00
0.50000E 06	0.60330E 00
0.60000E 06	0.60638E 00
0.70000E 06	0.60894E 00
0.80000E 06	0.61114E 00
0.90000E 06	0.61306E 00

TABLE 2

Tabulated solutions for p_D versus t_D'/r_D^2 for a single constant pressure well in an infinite system including wellbore skin effect, from -20 to +20

r_D = 1000

0.10000E-01	0.97375E-06
0.20000E-01	0.10682E-05
0.30000E-01	0.63597E-05
0.40000E-01	0.89804E-05
0.50000E-01	0.84462E-04
0.60000E-01	0.28038E-03
0.70000E-01	0.61620E-03
0.80000E-01	0.10918E-02
0.90000E-01	0.16964E-02
0.10000E 00	0.24146E-02
0.20000E 00	0.12820E-01
0.30000E 00	0.24227E-01
0.40000E 00	0.34556E-01
0.50000E 00	0.43631E-01
0.60000E 00	0.51606E-01
0.70000E 00	0.58678E-01
0.80000E 00	0.64994E-01
0.90000E 00	0.70679E-01
0.10000E 01	0.75852E-01
0.20000E 01	0.11089E 00
0.30000E 01	0.13148E 00
0.40000E 01	0.14589E 00
0.50000E 01	0.15689E 00
0.60000E 01	0.16575E 00
0.70000E 01	0.17315E 00
0.80000E 01	0.17948E 00
0.90000E 01	0.18500E 00
0.10000E 02	0.18989E 00
0.20000E 02	0.22087E 00
0.30000E 02	0.23803E 00
0.40000E 02	0.24980E 00
0.50000E 02	0.25868E 00
0.60000E 02	0.26580E 00
0.70000E 02	0.27171E 00
0.80000E 02	0.27676E 00
0.90000E 02	0.28116E 00
0.10000E 03	0.28505E 00
0.20000E 03	0.30965E 00
0.30000E 03	0.32329E 00
0.40000E 03	0.33265E 00
0.50000E 03	0.33973E 00
0.60000E 03	0.34540E 00
0.70000E 03	0.35013E 00
0.80000E 03	0.35417E 00
0.90000E 03	0.35769E 00

0.10000E 04	0.36081E 00
0.20000E 04	0.38059E 00
0.30000E 04	0.39161E 00
0.40000E 04	0.39919E 00
0.50000E 04	0.40495E 00
0.60000E 04	0.40957E 00
0.70000E 04	0.41342E 00
0.80000E 04	0.41671E 00
0.90000E 04	0.41959E 00
0.10000E 05	0.42214E 00
0.20000E 05	0.43837E 00
0.30000E 05	0.44745E 00
0.40000E 05	0.45372E 00
0.50000E 05	0.45848E 00
0.60000E 05	0.46231E 00
0.70000E 05	0.46551E 00
0.80000E 05	0.46825E 00
0.90000E 05	0.47064E 00
0.10000E 06	0.47277E 00
0.20000E 06	0.48632E 00
0.30000E 06	0.49393E 00
0.40000E 06	0.49919E 00
0.50000E 06	0.50320E 00
0.60000E 06	0.50643E 00
0.70000E 06	0.50913E 00
0.80000E 06	0.51144E 00
0.90000E 06	0.51346E 00

TABLE 3

Tabulated solutions for p_D versus t_D'/r_D^2 for a single constant pressure well in an infinite system including wellbore skin effect, from -20 to +20

$r_D = 10000$

0.10000E-01	0.24037E-06
0.20000E-01	0.75214E-05
0.30000E-01	0.53128E-05
0.40000E-01	0.61157E-05
0.50000E-01	0.55069E-04
0.60000E-01	0.18652E-03
0.70000E-01	0.41579E-03
0.80000E-01	0.74431E-03
0.90000E-01	0.11657E-02
0.10000E 00	0.16698E-02
0.20000E 00	0.91592E-02
0.30000E 00	0.17565E-01
0.40000E 00	0.25285E-01
0.50000E 00	0.32137E-01
0.60000E 00	0.38205E-01
0.70000E 00	0.43619E-01
0.80000E 00	0.48479E-01
0.90000E 00	0.52873E-01
0.10000E 01	0.56887E-01
0.20000E 01	0.84426E-01
0.30000E 01	0.10088E 00
0.40000E 01	0.11252E 00
0.50000E 01	0.12146E 00
0.60000E 01	0.12870E 00
0.70000E 01	0.13477E 00
0.80000E 01	0.13999E 00
0.90000E 01	0.14455E 00
0.10000E 02	0.14860E 00
0.20000E 02	0.17453E 00
0.30000E 02	0.18907E 00
0.40000E 02	0.19911E 00
0.50000E 02	0.20675E 00
0.60000E 02	0.21288E 00
0.70000E 02	0.21799E 00
0.80000E 02	0.22238E 00
0.90000E 02	0.22620E 00
0.10000E 03	0.22959E 00
0.20000E 03	0.25119E 00
0.30000E 03	0.26329E 00
0.40000E 03	0.27164E 00
0.50000E 03	0.27799E 00
0.60000E 03	0.28310E 00
0.70000E 03	0.28736E 00
0.80000E 03	0.29101E 00
0.90000E 03	0.29420E 00

0.10000E 04	0.29703E 00
0.20000E 04	0.31510E 00
0.30000E 04	0.32524E 00
0.40000E 04	0.33226E 00
0.50000E 04	0.33761E 00
0.60000E 04	0.34191E 00
0.70000E 04	0.34551E 00
0.80000E 04	0.34859E 00
0.90000E 04	0.35128E 00
0.10000E 05	0.35368E 00
0.20000E 05	0.36899E 00
0.30000E 05	0.37762E 00
0.40000E 05	0.38360E 00
0.50000E 05	0.38815E 00
0.60000E 05	0.39183E 00
0.70000E 05	0.39490E 00
0.80000E 05	0.39754E 00
0.90000E 05	0.39985E 00
0.10000E 06	0.40190E 00
0.20000E 06	0.41504E 00
0.30000E 06	0.42246E 00
0.40000E 06	0.42762E 00
0.50000E 06	0.43155E 00
0.60000E 06	0.43472E 00
0.70000E 06	0.43738E 00
0.80000E 06	0.43966E 00
0.90000E 06	0.44166E 00

APPENDIX - 3

TABLES

TABLE 4

Reservoir Rock and Fluid Data for Example 1

p_i = 3000 psi

p_{w_f} = 1400 psi

r_w = 0.5 ft

h = 36 ft

B_o = 1.315 res. bbl/STB

c_t = 10 x 10^{-6} psi^{-1}

μ = 3.1 cp

ϕ = 0.2

t (hrs)	q (STB)	1/q (STB^{-1})
0.01	980	1.02 x 10^{-3}
0.02	952	1.05 x 10^{-3}
0.05	893	1.12 x 10^{-3}
0.10	800	1.25 x 10^{-3}
0.50	694	1.44 x 10^{-3}
0.80	649	1.54 x 10^{-3}
1.00	625	1.60 x 10^{-3}
2.00	588	1.70 x 10^{-3}
5.00	546	1.83 x 10^{-3}
10.00	526	1.90 x 10^{-3}
20.00	490	2.04 x 10^{-3}
50.00	461	2.17 x 10^{-3}
90.00	446	2.24 x 10^{-3}
100.00	439	2.28 x 10^{-3}

TABLE 5

Reservoir Rock and Fluid Data for Example 2

WELL A - Constant Pressure Producer

$h = 60.4$ ft

$\phi = 0.25$

$B = 1.313$ res bbl/STB

$\mu = 3$ cp

$C_t = 13 \times 10^{-6}$ psi^{-1}

$r_w = 0.6$ ft

$p_i = 3000$ psi

$p_{w_f} = 1000$ psi

t (hrs)	q (STB/DAY)	1/q (DAY/STB)
0.017	884	1.131×10^{-3}
0.167	720	1.389×10^{-3}
0.333	680	1.471×10^{-3}
1.000	625	1.600×10^{-3}
5.000	570	1.754×10^{-3}
10.000	536	1.866×10^{-3}
20.000	516	1.938×10^{-3}
50.000	487	2.053×10^{-3}
90.000	467	2.141×10^{-3}
100.000	463	2.160×10^{-3}
120.000	458	2.183×10^{-3}
150.000	453	2.208×10^{-3}

TABLE 6

Pressure Interference Data for Observation Well B in Example 2

WELL B - Observation Well, 300 ft from the producer.

LOCATION: x = 300, y = 0

p_i = 3000 psi

t (hrs)	p(r,t)psi	Δp(psi)
0.017	----	----
0.167	2944	56
0.333	2882	118
1.000	2764	236
5.000	2576	424
10.000	2504	496
20 000	2440	560
50.000	2350	650
90.000	2310	690
100.000	2290	710
120.000	2280	720
150.000	2270	730

TABLE 7

Pressure Interference Data for Observation Well C in Example 2

WELL C - Observation Well, 180 ft from the producer.

LOCATION: $x = -125$, $y = 125$

$p_i = 3000$ psi

t(hrs)	p(r,t)psi	Δp(psi)
0.017	-----	----
0.167	2848	152
0.333	2776	224
1.000	2640	360
5.000	2438	562
10.000	2380	620
20.000	2310	690
50.000	2244	756
90.000	2200	800
100.000	2190	810
120.000	2180	820
150.000	2160	840

APPENDIX - 4

COMPUTER PROGRAMS

```
C     ANALYTICAL SOLUTIONS FOR CONSTANT WELL BORE PRESSURE
C     ====================================================
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/PARA/SKIN,RDIM,REFF,TFLOW
      COMMON/TSOLN/ICHART,NSOLN,ITYPE,IXA,IXB
      COMMON/HB/G1,G2,G3,G4,G5
      COMMOM/VAR/QD(1000),TD(1000),TDX(100),PD(50,100),
     &PD1(50,100),PD2(50,100),PDL(50,100)
      COMMON/RR/RD1,RD2
      CHARACTER*4 SIGN
      DOUBLE PRECISION TFORM,TFORMA,TFORMB,BESK0,BESK1
      DOUBLE PRECISION TFORMP
      DOUBLE PRECISION BESI0,BESI1,EI
      EXTERNAL TFORM,TFORMA,TFORMB
      EXTERNAL TFORMP
      PRINT,'READ IN N'
      READ,N
      M=0
      SIGN=' '
C
C
C     SOLUTION DESCRIPTION:
C     ====================
C
C
C     ICHART = 1 FOR QD VS TD
C              2 FOR QD VS TDA
C              4 FOR PD VS RD
C              7 FOR PRESSURE PROFILE IN THE RESERVOIR P(X,Y).
C
C
      PRINT,'READ IN THE VALUE FOR ICHART'
      READ,ICHART
C     LIMITS FOR TD ARE 10**IXA TO 10**IXB
      PRINT,'READ IN IXA'
      READ,IXA
      PRINT,'READ IN IXB'
      READ,IXB
C     NSOLN =  1 FOR INFINITE OUTER BOUNDARY
C              2 FOR NO-FLOW OUTER BOUNDARY
C              3 FOR CONSTANT PRESSURE OUTER BOUNDARY
      NSOLN=1
```

```
C     NTIMES = NUMBER OF LOG CYCLES TO EVALUATE
      NTIMES=(IXB-IXA)
C
C
C     PARAMETER VALUES:
C     ================
C
C
C     SKIN = WELLBORE SKIN FACTOR
      PRINT,'READ IN THE VALUE FOR SKIN'
      READ,SKIN
C     RDIM = DIMENSIONLESS RADIUS (1 .LE. RDIM .LE. REFF)
      PRINT,'READ IN RDIM'
      READ,RDIM
      AL=RDIM/2
C     REFF = DIMENSIONLESS RESERVOIR RADIUS (FOR FINITE RESERVOIR)
      REFF=50.
C     TFLOW = FLOW TIME (FOR PRESSURE BUILDUP OR PD VS RD)
      TFLOW=10.
C     NRD = NUMBER OF RADIAL LOG CYCLES
      NRD=5
C     BD = DIMENSIONLESS WIDTH OF LINEAR FLOW CHANNEL, B/RW
      BD=10.
      IF (ICHART .LT. 3) GO TO 5
      IF (ICHART .EQ. 4) GO TO 30
C
C
C     CALCULATE TIMES FOR EVALUATION.
C     ===============================
C
C
    5 T0=3.14159 *REFF*REFF*0.1
      TMULT=1.
      DLOGT=1./NTIMES
      IF (ICHART .EQ. 2) TMULT=REFF*REFF
      IF (ICHART .EQ. 5) TMULT = BD*BD
      DO 10 J=1,NTIMES
      DO 10 I=1,9
      K=I+(J-1)*9
      TD(K)=I*10.**(IXA+J-1)
   10 TDX(K)=TD(K)/TMULT
   11 CALL OUTFORM
C
C
```

```
C     CALCULATE QD.
C     =============
C
C
      NT=9*NTIMES
      IF(ICHART.EQ.7) GO TO 400
      DO 20 I=1,NT
      TDI=TD(I)
      IF ((TDI .GT. TO) .AND. (NSOLN .EQ. 2)) SIGN='*'
      CALL LINV(TFORM,TDI,QDI,N,M)
      Q=1/QDI
      WRITE(10,700)TDX(I),QDI
  700 FORMAT(2(E12.5))
      PRINT 300, TDX(I),QDI
   20 QD(I)=QDI
      GO TO 50
   30 CALL OUTFORM
C
C
C     CALCULATE PD VS RD FOR TD = TFLOW
C     =================================
C
C
      DO 40 J=1,NRD
      DO 40 I=1,9
      K=I+(J-1)*9
      RDIM=I*10.**(J-1)
      CALL LINV(TFORM,TFLOW,PD,N,M)
   40 WRITE (6,300) RDIM,PD
  300 FORMAT (' ',1PE10.2,2X,2(1PE12.4,2X),A1)
   50 STOP
  400 CONTINUE
      WRITE(10,1000)SKIN,RDIM,AL
 1000 FORMAT(2X,' S = ',E12.5,' RD = ',E12.5,' L = ',E12.5,
     &/,2X,54('#'),/,/,/)
      WRITE(10,2000)
 2000 FORMAT(2X,78('='),/,9X,'X',12X,'Y',9X,'PDL',10X,'PD',9X,
     &'PD1',9X,'PD2',/,2X,78('='),/,/,/)
      DO 430  I=1,NT,9
      TDI=TD(I)
      CALL LINV(TFORM,TDI,QDII,N,M)
      DO 420 JY=1,26
      DO 410 KX=1,51
      YY=JY*20-20
```

```
      XX=KX*20-20
      RD1=DSQRT((XX-AL)**2+YY**2)
      RD2=DSQRT((XX+AL)**2+YY**2)
      IF(RD1 .EQ. 0) GO TO 410
      CALL LINV(TFORMP,TDI,QDI,N,M,RD1,RD2)
      PD(JY,KX)=QDI
      PDL(JY,KX)=ALOG(PD(JY,KX))
      PD1(JY,KX)=QDI/QDII
      CALL PFORMA(TDI,PDI1,N,M,RD1)
      CALL PFORMA(TDI,PDI2,N,M,RD2)
      PD2(JY,KX)=PDI1+PDI2
  410 CONTINUE
  420 CONTINUE
      CALL PRINTPD(PD,TDI,PD1,PD2,PDL)
      PRINT,' '
      PRINT,' '
      PRINT,' ITS WORKING'
      PRINT,' '
      PRINT,' '
      PRINT,' '
  430 CONTINUE
      STOP
      END
C
C
C
C
      SUBROUTINE LINV(P,T,FA,N,M)
C     ===========================
C
C
C     Subroutine LINV (laplace Inverter)is a fortran translation
C     of the ALGOL procedure given by Stehfest (1970). P is  the
C     laplace space expression to be numerically inverted . T is
C     the time at which the solution is to be evaluated . FA  is
C     the value  of  the solution at time  T , determined by the
C     numerical  inversion  of the laplace space solution . N is
C     the number of terms in the summation .
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/LPL/G(50),V(50),H(25),GZ(1)
      DOUBLE PRECISION P
      DLOGTW=.6931471805599453
```

```
      IF (M .EQ. N) GO TO 100
C     CALCULATE V ARRAY.
      M=N
      G(1)=1.
      NH=N/2
      DO 5 I=2,N
    5 G(I)=G(I-1)*I
      H(1)=2./G(NH-1)
      DO 10 I=2,NH
      FI=I
      IF (I .EQ. NH) GO TO 8
      H(I)=FI**NH*G(2*I)/(G(NH-I)*G(I)*G(I-1))
      GO TO 10
    8 H(I)=FI**NH*G(2*I)/(G(I)*G(I-1))
   10 CONTINUE
      SN=2*(NH-NH/2*2)-1
      DO 50 I=1,N
      V(I)=0.
      K1=(I+1)/2
      K2=I
      IF (K2 .GT. NH) K2=NH
      DO 40 K=K1,K2
      IF (2*K-I .EQ. 0) GO TO 37
      IF (I .EQ. K) GO TO 38
      V(I)=V(I)+H(K)/(G(I-K)*G(2*K-I))
      GO TO 40
   37 V(I)=V(I)+H(K)/G(I-K)
      GO TO 40
   38 V(I)=V(I)+H(K)/G(2*K-I)
   40 CONTINUE
      V(I)=SN*V(I)
      SN=-SN
   50 CONTINUE
  100 FA=0.
      A=DLOGTW/T
      DO 110 I=1,N
      ARG=I*A
  110 FA=FA+V(I)*P(ARG)
      FA=A*FA
      RETURN
      END
C
C
C
```

```
C
      DOUBLE PRECISION FUNCTION TFORM(S)
C     ==================================
C
C
C     Double  Precision  Function  TFORM  contains the laplace
C     transformed solutions for the transient rate decline due
C     to  two  constant pressure wells flowing in interference
C     with eachother .  RDIM  is  the distance between the two
C     wells .
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/PARA/SKIN,RDIM,REFF,TFLOW
      COMMON/TSOLN/ICHART,NSOLN,ITYPE,IXA,IXB
      COMMON/HB/G1,G2,G3,G4,G5
      DIMENSION ARG(3),XK(2,3),XI(2,3)
      EXTERNAL BESK0,BESK1,BESI0,BESI1
      DOUBLE PRECISION BESK0,BESK1,BESI0,BESI1
      TFORM=BESK1(DSQRT(S))/DSQRT(S)/(BESK0(DSQRT(S))+
     &SKIN*DSQRT(S)*BESK1(DSQRT(S))+BESK0(RDIM*DSQRT(S)))
      RETURN
      END
C
C
C
C
C
C
      DOUBLE PRECISION FUNCTION TFORMA(S)
C     ===================================
C
C
C     Double  Precision  FUnction  TFORMA  is the laplace space
C     solution for cumulative production .
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DOUBLE PRECISION TFORM
      EXTERNAL TFORM
      TFORMA=TFORM(S)/S
      RETURN
      END
C
```

```
C
C
C
      DOUBLE PRECISION FUNCTION TFORMB(S)
C     ===================================
C
C
C     Double Precision Function TFORMB is the laplace space
C     solution for transient wellbore pressure with constant
C     rate production .
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DOUBLE PRECISION TFORM
      EXTERNAL TFORM
      TFORMB=1./(S*S*TFORM(S))
      RETURN
      END
C
C
C
C
      SUBROUTINE PFORM(T,P,N,M,T0)
C     ============================
C
C
C     Subroutine  PFORM  uses  limiting forms of the wellbore
C     pressure solution for constant rate production whenever
C     required .
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/PARA/SKIN,RDIM,REFF,TFLOW
      COMMON/TSOLN/ICHART,NSOLN,ITYPE,IXA,IXB
      DOUBLE PRECISION TFORM,TFORMA,TFORMB,BESK0,BESK1
      EXTERNAL TFORM,TFORMA,TFORMB
      NCASE=3
      IF (T .LT. 0.01) GO TO 30
      IF (T .LT. 100.) NCASE=1
      IF (T .GT. T0) NCASE=2
      GO TO (10,20,22),NCASE
   20 GO TO (22,24,26),NSOLN
   22 P=.5*(DLOG(T)+.80907)+SKIN
      RETURN
```

```
   24 P=DLOG(REFF)-.75+2.*T/(REFF*REFF)+SKIN
      RETURN
   26 P=DLOG(REFF)+SKIN
      RETURN
   10 CALL LINV(TFORMB,T,P,N,M)
      RETURN
   30 P=DSQRT(4.*T/3.1416)
      RETURN
      END
C
C
C
C
      SUBROUTINE QFORM (T,Q,N,M,T1)
C     =============================
C
C
C     Subroutine  QFORM  uses  limiting forms for rate decline
C     for constant pressure production whenever required .
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/PARA/SKIN,RDIM,REFF,TFLOW
      COMMON /TSOLN/ICHART,NSOLN,ITYPE,IXA,IXB
      DOUBLE PRECISION TFORM,TFORMA,TFORMB
      EXTERNAL TFORM,TFORMA, TFORMB
      NCASE=1
      IF (T .LT. 5.D04) NCASE=1
      IF (T .GT. T1) NCASE=2
   20 GO TO (22,24,26),NCASE
   22 Q=2./((DLOG(T)+.80907)+SKIN)
      RETURN
   24 Q=DEXP(-.1*T/T1)/(DLOG(REFF)-.75+SKIN)
      RETURN
   26 T2=2.*T1
      IF(T .LT. T2) GO TO 10
      Q=1./(DLOG(REFF)+SKIN)
      RETURN
   10 CALL LINV(TFORM,T,Q,N,M)
      RETURN
      END
C
C
C
```

```
C
      SUBROUTINE OUTFORM
C     ==================
C
C
C     Subroutine  OUTFORM  is used to obtain formated outputs
C     of  the  rate  transients  and inner and outer boundary
C     conditions .
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /PARA/SKIN,RDIM,REFF,TFLOW
      COMMON/TSOLN/ICHART,NSOLN,ITYPE,IXA,IXB
      COMMON/HB/G1,G2,G3,G4,G5
      COMMON/VAR/QD(1000),TD(1000),TDX(100),AMODES(100)
      IF (NSOLN .EQ. 1) PRINT 100
      IF (NSOLN .EQ. 2) PRINT 101
      IF (NSOLN .EQ. 3) PRINT 102
      GO TO (10,20,10,30,40,10,41),ICHART
   10 PRINT 103,SKIN,RDIM
      IF (NSOLN .NE. 1) PRINT 110, REFF
      IF (RDIM .EQ. 1.) PRINT 104
      IF (RDIM .NE. 1.) PRINT 105
      RETURN
   20 PRINT 103, SKIN, RDIM
      IF (NSOLN .NE. 1) PRINT 110, REFF
      IF (RDIM .EQ. 1.) PRINT 106
      IF (RDIM .NE. 1.) PRINT 107
      RETURN
   30 PRINT 108, SKIN, TFLOW
      RETURN
   40 PRINT 103, SKIN, RDIM
      PRINT 111
      PRINT, 'LINEAR FLOW MODEL'
      RETURN
   41 PRINT 109,SKIN,RDIM,RDIM/2
      RETURN
  100 FORMAT ('1UNBOUNDED RESERVOIR')
  101 FORMAT ('1CLOSED BOUNDED RESERVOIR')
  102 FORMAT ('1CONSTANT PRESSURE BOUNDED RESERVOIR')
  103 FORMAT (' SKIN = ',F6.3,3X,'RD = ',E12.4)
  104 FORMAT (/,6X,'TD',11X,'QD')
  105 FORMAT (/,6X,'TD',11X,'PD')
  106 FORMAT (/,5X,'TDA',11X, 'QD')
```

```
  107 FORMAT (/,5X,'TDA',11X,'QD')
  108 FORMAT (' SKIN = ', F6.3,3X,'TD = ',E12.4,/,6X,'RD',11X,'PD')
  109 FORMAT('SKIN = ',F6.3,3X,'RD = ',E12.4,3X,'L = ',E12.4)
  110 FORMAT (' OUTER RADIUS, RD = ', E12.4)
  111 FORMAT (/,6X,'TD',10X,'TDB',11X,'PD')
      END
C
C
C
C
      DOUBLE PRECISION FUNCTION TFORMP(S)
C     ====================================
C
C
C     Double  Precision  Function  TFORMP  contains  the laplace
C     space solutions  of  the pressure profile in the reservoir
C     around two constant pressure wells flowing in interference
C     with eachother .
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/PARA/SKIN,RDIM,REFF,TFLOW
      COMMON/TSOLN/ICHART,NSOLN,ITYPE,IXA,IXB
      COMMON/HB/G1,G2,G3,G4,G5
      COMMON/RR/RD1,RD2
      DIMENSION ARG(3),XK(2,3),XI(2,3)
      EXTERNAL BESK0,BESK1,BESI0,BESI1,TFORM
      DOUBLE PRECISION BESK0,BESK1,BESI0,BESI1,TFORM
      TFORMP=(BESK0(RD1*DSQRT(S))+BESK0(RD2*DSQRT(S)))/
     &(S*(BESK0(DSQRT(S))))
      RETURN
      END
C
C
C
C
      SUBROUTINE PRINTPD(PD,TDI,PD1,PD2,PDL)
C     ======================================
C
C
C     Subroutine  PRINTPD  develops the output of the pressure
C     profile in the reservoir at each specified node X and Y .
C
C
```

```
      DOUBLE PRECISION PD(50,100),TDI,PD1(50,100),PD2(50,100)
      WRITE(10,300)TDI
  300 FORMAT(2X,' TD = ',E12.5,/,2X,17('='),/)
      DO 10 JY=1,26
      DO 10 KX=1,51
      YY=JY*20-20
      XX=KX*20-20
      WRITE(11,200)YY,XX,PDL(JY,KX),PD(JY,KX),PD1(JY,KX)
  200 FORMAT(6(4X,E12.5))
   10 WRITE(10,100)XX,YY,PDL(JY,KX),PD(JY,KX),PD1(JY,KX)
  100 FORMAT(6(2X,E12.5))
      RETURN
      END
C
C
C
C
      DOUBLE PRECISION FUNCTION BESI0 (X)
C     ===================================
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION TT(12)
      ISIZE = 0
      IF(DABS(X) .GT. 80.) GO TO 20
      IF (DABS(X) .GT. 3.75) ISIZE = 1
      T = X/3.75
      IF (ISIZE .EQ. 1) T=1./DABS(T)
      TT(1) = T
      DO 1 I=2,12
    1 TT(I)=TT(I-1)*T
      IF (ISIZE .EQ. 1) GO TO 10
      BESI0 = 1. +3.5156229*TT(2) + 3.0899424*TT(4)
     &+1.2067492*TT(6) + 0.2659732*TT(8)
     &+0.0360768*TT(10) + 0.0045813*TT(12)
      RETURN
   10 BESI0 = (0.39894228 + 0.01328592*TT(1)
     &+0.00225319*TT(2) - 0.00157565*TT(3)
     &+0.00916281*TT(4) - 0.02057706*TT(5)
     &+0.02635537*TT(6) - 0.01647633*TT(7)
     &+0.00392377*TT(8))*DEXP(DABS(X))/DSQRT(DABS(X))
      RETURN
   20 BESI0=1.D38
      RETURN
```

```
      END
C
C
C
C
      DOUBLE PRECISION FUNCTION BESI1(X)
C     ===================================
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION TT(12)
      ISIZE=0
      IF (DABS(X) .GT. 80.) GO TO 20
      IF (DABS(X) .GT. 3.75) ISIZE = 1
      T=X/3.75
      IF (ISIZE .EQ. 1) T=1./DABS(T)
      TT(1)=T
      DO 1 I=2,12
    1 TT(I)=TT(I-1)*T
      IF(ISIZE .EQ. 1) GO TO 10
      BESI1 = (0.5+0.87890594*TT(2)+0.51498869*TT(4)
     &+0.15084934*TT(6)+0.2658733*TT(8)
     &+0.00301532*TT(10)+0.00032411*TT(12))*DABS(X)
      RETURN
   10 BESI1 = (0.39894228 - 0.03988024*TT(1)
     &-0.00362018*TT(2) + 0.00163801*TT(3)
     &-0.01031555*TT(4) + 0.02282967*TT(5)
     &-0.02895312*TT(6) + 0.0178654*TT(7)
     &-0.00420059*TT(8))*DEXP(DABS(X))/SQRT(DABS(X))
      RETURN
   20 BESI1=1.D38
      RETURN
      END
C
C
C
      DOUBLE PRECISION FUNCTION BESK0 (X)
C     ===================================
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DOUBLE PRECISION BESI0
      EXTERNAL BESI0
      DIMENSION TT(12)
```

```
      ISIZE = 0
      IF (DABS(X) .GT. 80.) GO TO 20
      IF (DABS(X) .GT. 2.) ISIZE = 1
      T = 0.5*DABS(X)
      IF (ISIZE .EQ. 1) T = 1./T
      TT(1)=T
      DO 1 I=2,12
    1 TT(I)=TT(I-1)*T
      IF (ISIZE .EQ. 1) GO TO 10
      BESK0 = -DLOG(T)*BESI0(X)-.57721566
     &+.42278420*TT(2) + .23069756*TT(4)
     &+.03488590*TT(6) + .00262698*TT(8)
     &+.00010750*TT(10)+.00000740*TT(12)
      RETURN
   10 BESK0= (1.25331414 - .07832358* TT(1)
     &+.02189568*TT(2)-.01062446*TT(3)
     &+.00587872*TT(4)-.00251540*TT(5)
     &+.00053208*TT(6))/DEXP(DABS(X))/DSQRT(DABS(X))
      RETURN
   20 BESK0=0.
      RETURN
      END
C
C
C
C
      DOUBLE PRECISION FUNCTION BESK1 (X)
C     ===================================
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DOUBLE PRECISION BESI1
      EXTERNAL BESI1
      DIMENSION TT(12)
      ISIZE = 0
      IF (DABS(X) .GT. 80.) GO TO 20
      IF (DABS(X) .GT. 2.) ISIZE = 1
      T = 0.5*DABS(X)
      IF(ISIZE .EQ. 1) T=1./T
      TT(1)= T
      DO 1 I=2,12
    1 TT(I)=TT(I-1)*T
      IF(ISIZE .EQ. 1) GO TO 10
      BESK1 = DLOG(T)*BESI1(X)+(1.+.15443144*TT(2)
```

```
     &-.67278579*TT(4)-.18156897*TT(6)
     &-.01919402*TT(8)-.00110404*TT(10)-.00004686*TT(12))/DABS(X)
      RETURN
   10 BESK1 = (1.25331414+.23498619*TT(1)-.03655620*TT(2)
     &+.01504268*TT(3)-.00780353*TT(4)+.00325614*TT(5)
     &-.00068245*TT(6))/DEXP(DABS(X))/SQRT(DABS(X))
      RETURN
   20 BESK1=0.
      RETURN
      END
```

```
REM           NEWTON RAPHSON ITERATION.
REM           ========================
REM
REM
REM   BASIC program to solve for the three permeability
REM   tensors using Newton Raphson Iteration.
REM
REM
  DIM X(3),Y(3),T9(2),T(2),F(3),K(3),K9(3),P(3,3),P1(3,3),D(3)
  PRINT "READ IN BIG X"
  MAT INPUT X
  PRINT "READ IN BIG Y"
  MAT INPUT Y
  PRINT "READ IN THE TIME VALUE FOR MATCH"
  MAT INPUT T
  PRINT "READ IN THE VALUE OF TD'/RD**2 FOR MATCH"
  MAT INPUT T9
  PRINT "READ IN KBAR"
  INPUT Z
  PRINT "READ IN 0.0002637/(PHI*MU*CSUBT*E^-2S) PRODUCT"
  INPUT K1
  K9(1)=K1*T(1)
  K9(2)=K1*T(2)
  K9(3)=1
  K(1)=30
  K(2)=20
  K(3)=13
  K8=(K(1)*K(2)-K(3)^2)
  FOR I=1 TO 2
  F(I)=-(K9(I)*K8/(K(1)*Y(I)^2+K(2)*X(I)^2-2*K(3)*X(I)*Y(I)))
 &+T9(I)
  NEXT I
  F(3)=-(K9(3)*K8/(K(1)*Y(3)^2+K(2)*X(3)^2-2*K(3)*X(3)*Y(3)))+Z
  FOR I=1 TO 3
  P3=(K(2)^2*X(I)^2-2*K(3)*K(2)*X(I)*Y(I)+K(3)^2*Y(I)^2)
  P4=(K(1)*Y(I)^2+K(2)*X(I)^2-2*K(3)*X(I)*Y(I))^2
  P(I,1)=K9(I)*(P3/P4)
  NEXT I
  FOR I=1 TO 3
  P5=(K(1)^2*Y(I)^2-2*K(1)*K(3)*X(I)*Y(I)+K(3)^2*X(I)^2)
  P6=(K(1)*Y(I)^2+K(2)*X(I)^2-2*K(3)*X(I)*Y(I))^2
  P(I,2)=K9(I)*(P5/P6)
  NEXT I
  FOR I=1 TO 3
  P7=(-2*K(3)*K(1)*Y(I)^2-2*K(3)*K(2)*X(I)^2+2*K(3)^2*X(I)
```

```
&*Y(I))
 P8=(2*K(1)*K(2)*X(I)*Y(I))
 P9=(K(1)*Y(I)^2+K(2)*X(I)^2-2*K(3)*X(I)*Y(I))^2
 P(I,3)=K9(I)*((P7+P8)/P9)
 NEXT I
 MAT P1=INV(P)
 MAT D=P1*F
 PRINT " P - MATRIX"
 MAT PRINT P
 PRINT "-----------------------------------------------------"
 PRINT" "
 PRINT " P1 - INV. MATRIX"
 MAT PRINT P1
 PRINT "-----------------------------------------------------"
 PRINT " F - FUNCTION"
 MAT PRINT F
 PRINT "-----------------------------------------------------"
 PRINT " D - DEL"
 MAT PRINT D
 PRINT "-----------------------------------------------------"
 PRINT " "
 MAT K=K+D
 E=0
 PRINT " "
 PRINT " KXX = "K(1)
 PRINT " KYY = "K(2)
 PRINT " KXY = "K(3)
 PRINT " "
 FOR I=1 TO 3
 E=E+ABS(D(I)/K(I))
 PRINT " "
 PRINT " E = "E
 PRINT " "
 NEXT I
 PRINT "====================================================="
 IF E > 0.0000001 GO TO 200
 PRINT " KXX , KYY , KXY"
 MAT PRINT K
 STOP
 END
```

```
C        PRESSURE PROFILE AROUND CONSTANT RATE WELLS.
C        ============================================
C
C
C     This program calculates the pressure profile in the
C     reservoir around two constant rate wells flowing in
C     interference with eachother.
C
C
      REAL PP(100,100),PPL(100,100),PQ(100,100),PQL(100,100)
      REAL K,MU,CT,PHI,AL,Q,T,RDIM,PI,PWF,B,H
      PRINT,'READ IN THE FOLLOWING VALUES'
      PRINT,'K , MU , PHI , CSUBT , RSUBD , TIME , PI , PWF,
     & Q , B , H'
      PRINT,'==================================================
     &==========='
      READ , K,MU,PHI,CT,RD,T,PI,PWF,Q,B,H
      PRINT,' '
      AL=RD/2
      DO 20 JY=1,26
      DO 10 KX=1,51
      Y=JY*20-20
      X=KX*20-20
      RD1=SQRT((X-AL)**2+Y**2)
      RD2=SQRT((X+AL)**2+Y**2)
      TD1=(0.000264*K*T)/(PHI*MU*CT*(RD1**2))
      TD2=(0.000264*K*T)/(PHI*MU*CT*(RD2**2))
      PD1=0.5*(ALOG(TD1)+0.80907)
      PD2=0.5*(ALOG(TD2)+0.80907)
      PD=PD1+PD2
      PP(JY,KX)=(141.2*Q*B*MU*PD)/(K*H)
      PQ(JY,KX)=PI-PP(JY,KX)
      WRITE(10,100)X,Y,PQ(JY,KX)
  100 FORMAT(3(2X,E12.5))
   10 CONTINUE
   20 CONTINUE
      STOP
      END
```